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The Application of Coefficient Diagram Method to ACC Benchmark Problem

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Abstract

A controller design method, called Coefficient Diagram Method (CDM), is introduced. By this method the designer can design the characteristic polynomial of the closed loop system efficiently taking a good balance of stability, response, and robustness. By CDM, a solution of the ACC benchmark problem is given, and solutions given by various researchers are compared. Theoretical analysis is made to clarify the robustness trade-off.

1. Introduction

The purpose of this paper is to show the effectiveness of a control design method called "Coefficient Diagram Method (CDM)" by solving the benchmark problem proposed by Wie [1] in American Control Conference (ACC).

The CDM is fairly new and not well-known, but its basic philosophy has been known in control community for more than 30 years [2] [3] [4] [5] [6] [7]. The idea has been successfully used in many fields of industry such as steel mill control [4], gas turbine control [7], and spacecraft attitude control [8] [9].

The coefficient diagram is a semi-log diagram where the coefficients of characteristic polynomial are shown in logarithmic scale in the ordinate and the numbers of power corresponding to each coefficient are shown in the abscissa, as shown in Fig. 1. The degree of convexity is a measure of stability. The general inclination of the curve is a measure of response speed. The variation of the shape of the curve is a measure of robustness. Thus the three major characteristics of control system, namely stability, response, and robustness are shown graphically in a single diagram, enabling the designer to make a balanced judgment in the course of his design.

This paper will first explain the basics of CDM. Then ACC benchmark problem is briefly introduced, and a solution is derived by CDM. After brief explanation of the theoretical analysis, this result is compared with the best design so far achieved[10].

2. Basics of CDM

Features of CDM

The features of CDM will be summarized as follows;

(1) CDM is is a control system design method, where the coefficient diagram is used as a vehicle to carry the necessary information.

(2) The CDM is an algebraic design method over polynomial ring.

(3) The theoretical background of CDM is the sufficient condition of stability by Lipatov [11]. The tradition of Kessler [4] standard form is inherited and improved.

Summary of the basic relation used in CDM

The characteristic polynomial P(s) is given as

$$P(s) = a_n s^n + \dots + a_1 s + a_0 = \prod_{i=0}^{n} a_i s^i \quad (1)$$

The stability index γ_i, the equivalent time constant τ, and the stability limit γ_i^* are given as

$$\gamma_i = a_i^2 / (a_{i+1} a_{i-1}), \quad i = 1 \sim n-1, \quad (2)$$

$$\tau = a_1 / a_0, \quad (3)$$

$$\gamma_i^* = 1/\gamma_{i+1} + 1/\gamma_{i-1}, \quad i = 1 \sim n-1, \ \gamma_n = \gamma_0 = \infty. \quad (4)$$

The coefficient a_i of the characteristic polynomial are related to γ_i and τ as

$$a_{i+1} / a_i = (a_j / a_{j-1}) / (\gamma_i \gamma_{i-1} \cdots \gamma_{j+1} \gamma_j), \quad i \geq j, \quad (5)$$

$$a_i = a_0 \tau^i / (\gamma_{i-1} \gamma_{i-2}^2 \cdots \gamma_2^{i-2} \gamma_1^{i-1}). \quad (6)$$

The stability condition for the 3rd and 4th order is derived from Routh-Hurwitz stability condition.

$$\gamma_i > \gamma_i^*, \quad i = 1 \sim n-1. \quad (7)$$

For the system higher than or equal to 5th order, the system is stable [11], if

$$\gamma_i > 1.12 \gamma_i^*, \quad i = 1 \sim n-1 \quad \text{for all i.} \quad (8)$$

And the system is unstable, if

$$\sqrt{\gamma_{i+1} \gamma_i} < 1, \quad i = 1 \sim n-2 \quad \text{for some i.} \quad (9)$$

The standard value of γ_i is given as

$$\gamma_{n-1} = \dots = \gamma_3 = \gamma_2 = 2, \quad \gamma_1 = 2.5. \quad (10)$$

The choice of $\gamma_3 = \gamma_2 = 2$, $\gamma_1 = 2.5$ is strongly recommended. But for $\gamma_{n-1} \sim \gamma_4$, the condition can be relaxed as

$$\gamma_i > 1.5 \gamma_i^*, \quad n-1 \geq i \geq 4. \quad (11)$$

These relations will be fully utilized in the CDM design.

Coefficient diagram

When a characteristic polynomial P(s) is given as

$$P(s) = 0.25\,s^5 + s^4 + 2\,s^3 + 2\,s^2 + s + 0.2, \qquad (12)$$

the coefficient diagram is shown in Fig. 1, where coefficient a_i, stability index γ_i, equivalent time constant τ, and stability limit γ_i^* are shown in one figure. The stability index can be graphically obtained as in Fig. 2a. When the curvature of a_i curve becomes large, the system becomes more stable corresponding to larger γ_i's as shown in Fig. 2b. When the curve a_i is left-end-down, the equivalent time constant τ is small and the response is faster as shown in Fig. 2c.

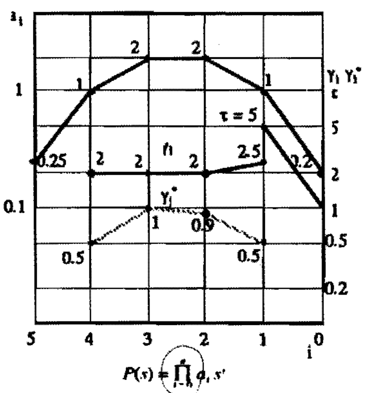
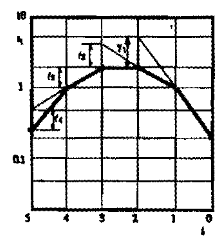


Fig. 1. Coefficient diagram



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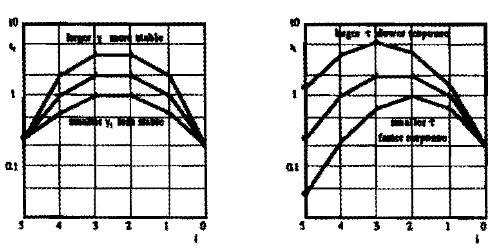

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Fig. 2. Explanation of coefficient diagram

3. Benchmark Problem

The bench mark problem [1] is the control of the two-mass-spring system shown in Fig. 3, which is a generic model of an uncertain dynamical system with a rigid-body mode and one vibration mode. It is assumed that for the nominal system $m_1 = m_2 = 1$ and $k = 1$ with appropriate units and time is in units of seconds. A control force, u, acts on body 1, and the position of body 2, x_2, is measured, resulting in a noncollocated actuator / sensor control problem. The disturbance forces are w_1 and w_2.

There are 4 problems in the benchmark problem, of which Problem 1 and 2 are considered here in order to clarify the nature of the problem. Because the specifications shown in the original problem have some ambiguity, Thompson [10] combined Problem 1 and 2, and made the specification more concrete.

The proposed problem (problem1 and 2) is as follows;

(1) For a unit impulse disturbance exerted on body 1 or body 2, the controlled output of the nominal system shall not exceed 0.1 after 15 time units.

(2) For the same disturbances the peak control level of the nominal system shall not exceed 1.

(3) The gain margin shall be 6 dB or greater and the phase margin shall be at least 30 deg.

(4) The closed-loop system shall be stable for $0.5 \leq k \leq 2.0$ and $m_1 = m_2 = 1$.

(5) The closed-loop system shall be stable for simultaneous changes $1 - pm \leq k, m_1, m_2 \leq 1 + pm$, $pm = 0.3$.

(6) There shall be reasonable high-frequency sensor noise rejection, performance robustness, and controller complexity.

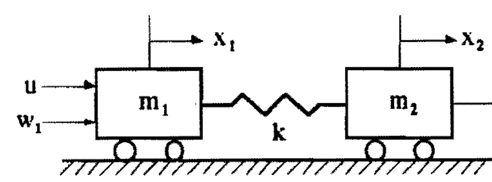

Fig. 3. Two-mass-spring system

4. Design by CDM

Preliminary design by CDM

The block diagram of the system is shown in Fig. 4. First design, CDM-1, is made using the standard value for γ_i's and the equivalent time constant $\tau = 6$, because settling time, 15 sec, is about $2.5\,\tau$.

$$\gamma_i = [2 \;\; 2 \;\; 2 \;\; 2 \;\; 2 \;\; 2.5], \quad \tau = 6. \qquad (13)$$

The result is shown in Table 1. Some shorthand notations are used as,

$$\gamma_i = [\gamma_6 \;\cdots\; \gamma_2 \;\; \gamma_1], \qquad a_i = [a_7 \;\cdots\; a_1 \;\; a_0],$$
$$k_i = [k_3 \;\; k_2 \;\; k_1 \;\; k_0], \qquad l_i = [l_3 \;\; l_2 \;\; l_1 \;\; l_0].$$

Also the key figure, KF, is defined as

$$KF = [\,k_3 / l_2 \quad k_2]. \qquad (14)$$

The first entry of the KF is a measure of the control effort and the second entry ia a measure of robustness. Also in order to summarize the results, the phase margin PM, the gain margin GM, the settling time t_s, the maximum value of the control effort u_{max}, the variation of k, k_{min} / k_{max}, and the simultaneous change limit pm are shown in a row vector FM (Figure of merit) defined as

$$FM = [PM \quad GM \quad t_s \quad u_{max} \quad k_{min} / k_{max} \quad pm]. \qquad (15)$$

The bold letters in FM indicate that the specification is not satisfied. Also score for FM, defined by Thompson [10], is shown. This design is robust and satisfy all the specification except the control effort u_{max}, which is fairly large. This design shows u_{max} and robustness is in a trade-off, and one has to be sacrificed for the other.

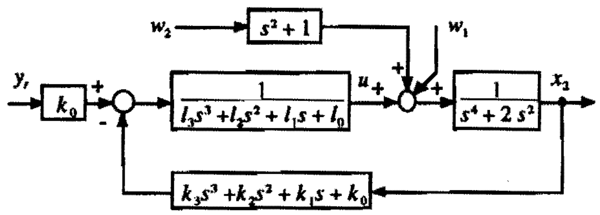

Fig. 4. Block diagram of control system

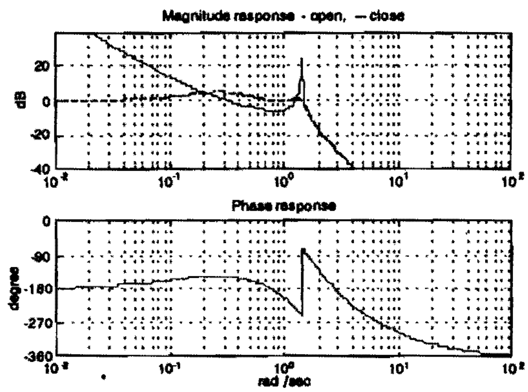

Fig. 5. Frequency response

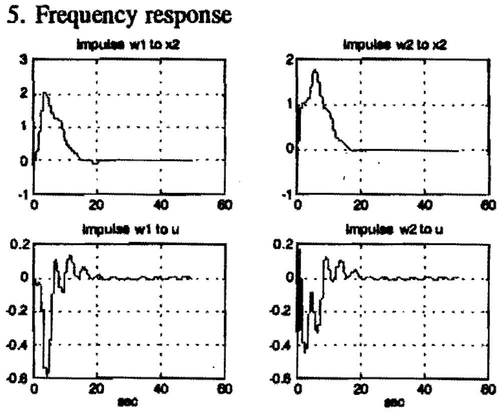

Fig. 6. Time response

Improvement of the control effort

In order to decrease u_{max}, it is necessary to make l_2 large and k_3 small. This can be achieved only by reducing the stability index γ_i, especially of the low order. The γ_2 and γ_1 can not be decreased, because then the time response will be deteriorated. Thus γ_5, γ_4, and γ_3 are decreased to 1.5. The lowest value considered is 1.5, because the sufficient condition for stability, Eq. (8), is always satisfied, if all γ_i's are larger than 1.5. The γ_6 is chosen as 2, because it does not affect l_2.

Thus CDM-2 design is made for

$$\gamma_i = [2 \quad 1.5 \quad 1.5 \quad 1.5 \quad 2 \quad 2.5], \quad \tau = 6. \qquad (16)$$

The result is shown in Table 1. This design satisfies u_{max} condition, but k_{min} / k_{max} is not satisfied. In order to find the cause for the instability, the stability index γ_i and the stability limit γ_i^* are compared for k variation case. By this analysis, it is found that the increase of γ_5 and γ_2 improves robustness.

Improvement of robustness

Now the design CDM-3 is made for

$$\gamma_i = [4 \quad 1.5 \quad 1.5 \quad 1.5 \quad 2.5 \quad 2], \quad \tau = 6. \qquad (17)$$

The result is shown in Table 1. This design satisfies all the specifications with comfortable margin.

Final design

Although CDM-3 is a satisfactory design, further improvement can be made by increasing τ, because the settling time has some margin. The final design is made for

$$\gamma_i = [4 \quad 1.5 \quad 1.5 \quad 1.5 \quad 2.5 \quad 2], \quad \tau = 6.4. \qquad (18)$$

The result is shown in Table 1. All the specifications are satisfied with comfortable margin and high score of 8.54 is achieved. The frequency responses of the open-loop transfer function are shown in Fig. 5, and the time responses are shown in Fig. 6.

5. Analysis of the Result

Theoretical analysis

The detailed theoretical analysis [15] reveals that, if GM $\geqq$ 6 dB is to be maintained, the bound of k_2 is approximately given as follows;

$$k_2 \geqq -0.31010. \qquad (19)$$

From this, the following condition can be derived.

$$\gamma_5 = \gamma_4 = \gamma_3 \leqq 1.4909. \qquad (20)$$

Also the condition $u_{max} \leqq 1$ gives

$$\gamma_5 = \gamma_4 = \gamma_3 \leqq 1.5022. \qquad (21)$$

From Eqs. (20) and (21), it will be safely concluded that γ_5, γ_4, and γ_3 must take the value around 1.5 in order to satisfy GM and u_{max} condition. Also it should be noted that the condition $\gamma_5 = \gamma_4 = \gamma_3 = 1.5$ satisfies the sufficient condition for stability Eq. (8). The trade-off analysis based on the above findings reveals the solution obtained by CDM-4 is almost "only" solution for the benchmark problem.

Comparison with published solutions

The Journal of Guidance, Control, and Dynamics (September-October 1992) presented 11 solutions to the problem. The first 6 solutions use various design techniques, and the following 5 solutions use a variety of techniques based on H∞ theory.

Thompson [10] made the specification more concrete and made his own design using classical / H_2 approach, where weights for H_2 solution are obtained from the classical control theory. By so doing, design procedure is more automated, while the rich experiences in classical control are retained. The result is shown in Table 1. It will be concluded his design is very similar to CDM-4.

Table 1 The CDM designs

CDM-1 Standard value design

γ_i = [2 2 2 2 2 2.5], τ = 6
k_i = [1.187 - 0.4738 0.6359 0.1060]
l_i = [0.003709 0.04945 0.3223 1]
KF = [1.187 / 0.04945 - 0.4738], score = - 1.20
FM = [39.5 6.6 12.9 **14.7** 0.35 / 3.25 0.39]

CDM-2 Reduced control effort design

γ_i = [2 1.5 1.5 1.5 2 2.5], τ = 6
k_i = [0.5126 - 0.3219 0.6992 0.1165]
l_i = [0.05431 0.3055 0.7506 1]
KF = [0.5126 / 0.3055 - 0.3219], score = - 0.65
FM = [38.2 6.2 12.8 0.97 **0.53 / 1.74** 0.23]

CDM-3 Robustness recovered design

γ_i = [4 1.5 1.5 1.5 2.5 2], τ = 6
k_i = [0.4040 - 0.3219 0.5594 0.09323]
l_i = [0.02716 0.3055 0.8049 1]
KF = [0.4040 / 0.3055 - 0.3219], score = 6.45
FM = [37.4 7.3 14.1 0.89 0.46 / 2.29 0.35]

CDM-4 Final design

γ_i = [4 1.5 1.5 1.5 2.5 2], τ = 6.4
k_i = [0.2039 - 0.3895 0.5033 0.07864]
l_i = [0.03599 0.3795 0.9287 1]
KF = [0.2039 / 0.3795 - 0.3895], score = 8.54
FM = [35.4 6.2 15.0 0.59 0.47 / 3.45 0.42]

Thompson [10] Eq. (19) Classical / H_2

γ_i = [1.929 2.627 1.563 1.552 1.451 2.543 2.020]
τ = 6.263
k_i = [0.3263 - 0.4548 0.4985 0.07960]
l_i = [0.002675 0.03914 0.2915 0.7787 1]
KF = [0.3263 / 0.2915 - 0.4548], score = 7.36
FM = [35.3 6.0 14.5 0.76 0.45 / 2.81 0.42]

6. Conclusion

The major results of this paper are as follows;

(1) The outline of the coefficient diagram method (CDM) is briefly explained.

(2) In order to evaluate the effectiveness of CDM, a controller for problem 1 and 2 of ACC benchmark problem is designed. The result is better than the best design of H_2 or H∞ control.

(3) By theoretical analysis, the trade-off among requirements is clarified. It is made clear that solution like CDM-4 or one by Thompson is the only possible solution to satisfy the requirement.

Although CDM is a powerful tool for control system design at this stage, further research is needed to make it effective to Multi-Input-Multi-Output system, and to make use of it for design of adaptive control systems.

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