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# Brief Tutorial and Survey of Coefficient Diagram Method

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**Abstract**

A tutorial of the Coefficient Diagram Method (CDM) will be given first and then the survey of CDM in recent years will be made. By CDM, the simplest controller to satisfy the specification can be designed efficiently. Its application ranges from ordinary PID control, mechanical system, and feed-forward control to sophisticated MIMO control in aerospace.

## 1 Introduction

With wide spread of control technology into various fields, simple and reliable control design approach is keenly needed. The classical control well answered to this need for the ordinary control design problems, but not for more complex plants. The modern control has been developed to answer to this need. However, in spite of the tremendous effort in the past, it has not reached to the satisfactory state because of following reasons.

(1) The controller parameter tuning is difficult.
(2) Theory is difficult for control non-specialists.
(3) Weight selection rules are not established.
(4) The controller is unnecessarily of high order.
(5) Sometimes an un-robust and fragile controller is designed for structured uncertainty.
(6) Sometimes good controller is excluded due to either deficiency of theory itself or its misinterpretation.
(7) The incorporation of gain-scheduling or non-linearity in controller is difficult.

The Coefficient Diagram Method (CDM) has been developed to answer this problem. The CDM is fairly new and it is not well-known, but its basic philosophy has been known in industry and in control community for more than 40 years [1][3] with successful application in servo control [1], steel mill drive control [3], gas turbine control [10], and spacecraft attitude control [5]. The historical background is given in [8]. In Section 2 and 3, tutorial of CDM is given. In Section 4, survey of recent development is made.

## 2 Basics of CDM

### 2.1 Basic Philosophy of CDM

The CDM is an algebraic control design approach [24] with the following five features [8].

(1) Polynomials and polynomial matrices are used for system representation.
(2) Characteristic polynomial and controller are simultaneously designed.
(3) Coefficient diagram is effectively utilized.
(4) The sufficient condition for stability by Lipatov constitutes the theoretical basis of CDM [5][9].
(5) Kessler standard form [3] is improved and used as the standard form of CDM.

CDM design is based on the stability index and equivalent time constant as defined later. Thus for the specified settling time, a controller of the lowest order with the narrowest bandwidth and of no-overshoot can be easily designed. CDM can be considered as “Generalized PID”, because the controller can be more complex than PID, and more reliable parameter selection rules are provided. Also CDM can be considered as “Improved LQG”, because the order of controller is smaller and weight selection rules are also given [7].

### 2.2 Mathematical model

The standard block diagram of the CDM design for a single-input-single-output system is shown in Fig. 1. The extension to multi-input-multi-output can be made with proper interpretation, but it is not discussed here for simplicity. The plant equation is given as

$$A_p(s)x = u + d \tag{1a}$$

$$y = B_p(s)x \, , \tag{1b}$$

where $u$, $y$, and $d$ are input, output, and disturbance. The symbol $x$ is called the basic state variable. $A_p(s)$ and $B_p(s)$ are the denominator and numerator polynomial of the plant transfer function $G_p(s)$. It will be easily seen that this expression has a direct correspondence with the control canonical form of the state-space expression, and $x$ corresponds to the state variable of the lowest order. All the other states are expressed as the derivatives of $x$ of high order. Controller equation is given as

$$A_c(s)u = B_a(s)y_r - B_c(s)(y + n)\, , \tag{2}$$

where $y_r$ and $n$ are reference input and noise on the output. $A_c(s)$ is the denominator of the controller transfer function. $B_a(s)$ and $B_c(s)$ are called the reference numerator and feedback numerator of the controller transfer function. Because the controller transfer function has two numerators, it is called two-degree-of-freedom system. This expression corresponds to the observer canonical form of the state-space expression. Elimination of $y$ and $u$ from Eq. (2) by Eqs. (1a, b) gives

$$P(s)x = B_a(s)y_r + A_c(s)d - B_c(s)n \, , \tag{3a}$$

where $P(s)$ is the characteristic polynomial and given as

$$P(s) = A_c(s)A_p(s) + B_c(s)B_p(s)\, . \tag{3b}$$

In a similar manner, equation for $y$ and $u$ can be obtained. Because this system has 3 inputs and 3 outputs, there are 9 transfer functions.

For CDM design, the following four basic relations are selected as standard, namely

$$P(s)x = P(0)y_r \quad (4a)$$

$$P(s)y = B_p(s)B_a(s)y_r \quad (4b)$$

$$P(s)y = B_p(s)A_c(s)d \quad (4c)$$

$$P(s)(-y) = B_p(s)B_c(s)n\,. \quad (4d)$$

Eq. (4a) is the response of $x$ to $y_r$ when $B_a(s) = P(0)$, and it corresponds to the canonical closed-loop transfer function of system type 1 for $P(s)$, which will be explained later. This equation specifies the characteristic polynomial, and it is a very good measure of stability. Eq. (4b) is for the command following characteristics. Eq. (4c) is for the disturbance rejection characteristics. Eq. (4d) corresponds to the complementary sensitivity function $T(s)$, and it is useful for checking the robustness. In the CDM design, these four basic relations are used as performance specification. The design of $P(s)$ is first made to satisfy specifications on Eqs. (4a)(4c)(4d), and then $B_a(s)$ is adjusted to satisfy the specification on Eq. (4b).

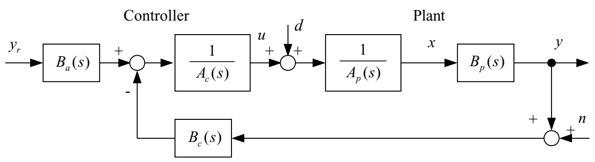


Fig. 1. Mathematical model

### 2.3 Mathematical relations

Some mathematical relations extensively used in CDM will be introduced hereafter. The characteristic polynomial is given in the following form.

$$P(s) = a_n s^n + ... + a_1 s + a_0 = \sum_{i=0}^{n} a_i s^i \quad (5)$$

The stability index $\gamma_i$, the equivalent time constant $\tau$, and stability limit $\gamma_i^*$ are defined as follows.

$$\gamma_i = a_1^2 / (a_{i+1} a_{i-1}), \qquad i = 1 \sim n-1 \quad (6a)$$

$$\tau = a_1 / a_0 \quad (6b)$$

$$\gamma_i^* = 1/\gamma_{i+1} + 1/\gamma_{i-1}$$
$$i = 1 \sim n-1, \quad \gamma_n = \gamma_0 = \infty \quad (6c)$$

Also the equivalent time constant of the i-th order $\tau_i$ is defined as follows.

$$\tau_i = a_{i+1} / a_i, \qquad i = 1 \sim n-1 \quad (7a)$$

Then the following relations are derived.

$$\tau_i = \tau_{i-1} / \gamma_i = \tau / (\gamma_i \cdots \gamma_2 \gamma_1) \quad (7b)$$

$$a_i = \tau_{i-1} ... \tau_2 \tau_1 \tau \ \ a_0 = a_0 \tau^i / \ (\gamma_{i-1} \gamma_{i-2}^2 \cdots \gamma_2^{i-2} \gamma_1^{i-1}) \quad (7c)$$

The characteristic polynomial will be expressed by $a_0$, $\tau$ and $\gamma_i$ as follows.

$$P(s) = a_0 [\{\sum_{i=2}^{n} (\prod_{j=1}^{i-1} 1/\gamma_{i-j}^{j})(\tau s)^i\} + \tau s + 1] \quad (8)$$

### 2.4 Coefficient diagram

When the plant/controller polynomials are given as

$$A_p(s) = 0.25s^4 + s^3 + 2s^2 + 0.5s, \quad B_p(s) = 1,$$
$$A_c(s) = l_1 s, \quad B_c(s) = k_2 s^2 + k_1 s + k_0, \quad (9)$$
$$l_1 = 1, \quad k_2 = 1.5, \quad k_1 = 1, \quad k_0 = 0.2,$$

the characteristic polynomial is expressed as

$$P(s) = 0.25s^5 + s^4 + 2s^3 + 2s^2 + s + 0.2\,. \quad (10)$$

Then

$$a_i = [a_5 \cdots a_2 \ a_1] = [0.25 \quad 1 \quad 2 \quad 2 \quad 1 \quad 0.2] \quad (11a)$$

$$\gamma_i = [\gamma_4 \cdots \gamma_2 \ \gamma_1] \ = [2 \quad 2 \quad 2 \quad 2.5] \quad (11b)$$

$$\tau = 5 \quad (11c)$$

$$\gamma_i^* = [\gamma_4^* \cdots \gamma_2^* \ \gamma_1^*] = [0.5 \quad 1 \quad 0.9 \quad 0.5] \quad (11d)$$

The coefficient diagram is shown as in Fig. 2, where coefficient $a_i$ is read by the left side scale, and stability index $\gamma_i$, equivalent time constant $\tau$, and stability limit $\gamma_i^*$ are read by the right side scale. The $\tau$ is expressed by a line connecting 1 to $\tau$. The stability index $\gamma_i$ can be graphically obtained (Fig. 3a). If the curvature of the $a_i$ becomes larger (Fig. 3a), the system becomes more stable, corresponding to larger stability index $\gamma_i$. If the $a_i$ curve is left-end down (Fig. 3b), the equivalent time constant $\tau$ is small and response is fast. The equivalent time constant $\tau$ specifies the response speed.

The coefficient diagram is also used for parameter sensitivity analysis and robustness analysis. In this example, the characteristic polynomial $P(s)$ is composed of two component polynomials: denominator polynomial $P_{l1}(s)$ and numerator polynomial $P_k(s)$.

$$P(s) = P_{l1}(s) + P_k(s) \quad (12a)$$

$$P_{l1}(s) = l_1(0.25s^5 + s^4 + 2s^3 + 0.5s^2) \quad (12b)$$

$$P_k(s) = k_2 s^2 + k_1 s + k_0 \quad (12c)$$

The auxiliary sensitivity function T($s$) is expressed as

$$T(s) = P_k(s) / P(s) \quad (12d)$$

Eq. (12b) is shown in Fig. 2 with small circles and dash-dot lines. Eq. (12c) is shown with small squares and dotted lines. Designer can visually assess the deformation of the coefficient diagram due to the parameter change of $k_2$, $k_1$, and $k_0$. Then he can visualize the variation of stability and response. Also from Eq. (12d), it is clear that robustness can be analyzed by comparison of coefficients $a_i$ and $k_i$ at the coefficient diagram.

Thus the coefficient diagram indicates stability, response, and robustness (three major properties in control design) in a single diagram, enabling the designer to grasp the total picture of control system. At present, Bode diagram is used for this purpose. However coefficient diagram is more accurate and easy to use in actual design.

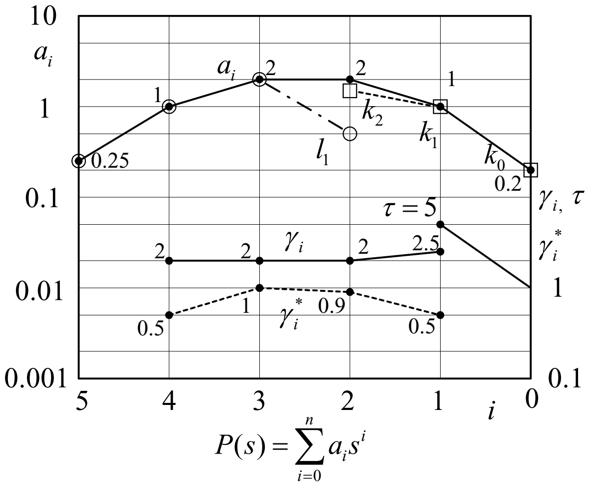


Fig. 2. Coefficient diagram

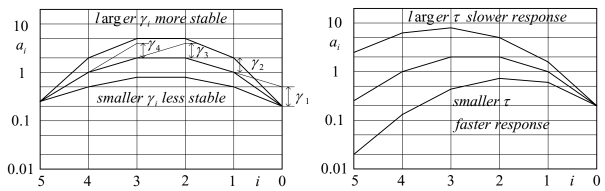


Fig. 3a. Effect of $\gamma_i$ b. Effect of $\tau$

## 2.5 Stability condition

From the Routh-Hurwitz stability criterion, the stability condition for the 3rd order is given as

$$a_2 a_1 > a_3 a_0 \,. \tag{13a}$$

If it is expressed by stability index,

$$\gamma_2 \gamma_1 > 1 \,. \tag{13b}$$

The stability condition for the fourth order is given as

$$a_2 > (a_1 / a_3) a_4 + (a_3 / a_1) a_0 \,, \tag{14a}$$

$$\gamma_2 > \gamma_2^* \,. \tag{14b}$$

For the system higher than or including 5th degree, Lipatov [4] gave the sufficient condition for stability and instability in several different forms. The conditions most suitable to CDM can be stated as follows;

"The system is stable, if all the partial 4th order polynomials are stable with the margin of 1.12. The system is unstable if some partial 3rd order polynomial is unstable."

Thus the sufficient condition for stability is given as

$$a_i > 1.12 \left[ \frac{a_{i-1}}{a_{i+1}} a_{i+2} + \frac{a_{i+1}}{a_{i-1}} a_{i-2} \right], \tag{15a}$$

$$\gamma_i > 1.12 \gamma_i^* \,, \quad \textit{for all } i = 2 \sim n-2 \,. \tag{15b}$$

The sufficient condition for instability is given as

$$a_{i+1} a_i \le a_{i+2} a_{i-1} \,, \tag{16a}$$

$$\gamma_{i+1} \gamma_i \le 1 \,, \quad \textit{for some } i = 1 \sim n-2 \,. \tag{16b}$$

These conditions can be graphically expressed in the coefficient diagram. Fig. 4a is a 3rd-order example. Point A is $(a_2 a_1)^{0.5}$ and point B is $(a_3 a_0)^{0.5}$. Thus if A is above B, the system is stable. Point C is $(\gamma_2 \gamma_1)^{0.5}$. If it is above 1, the system is stable. Fig. 4b is a 4th-order example. Point A is obtained by drawing a line from $a_4$ in parallel with line $a_3 a_1$. Similarly point B is obtained by drawing a line from $a_0$ in parallel with line $a_3 a_1$. The stability condition is $a_2 >$ (A + B). The other condition is $\gamma_2 > \gamma_2^*$.

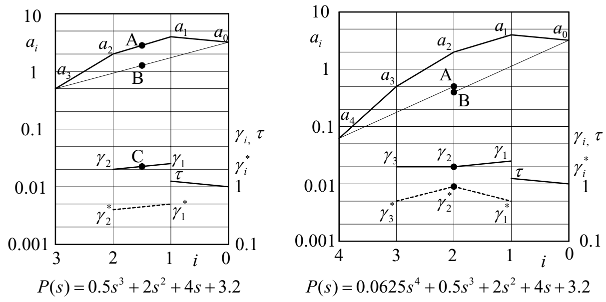


Fig. 4a. 3rd order b. 4th order

## 2.6 Canonical transfer function

For a given characteristic polynomial, there exist infinite number of open-loop and closed-loop transfer functions. The specific transfer functions to represent the characteristic polynomial, called canonical transfer function, are defined as follows.

System type 1, canonical open/closed-loop transfer function, $G_1(s)$ and $T_1(s)$:

$$G_1(s) = a_0 / (a_n s^n + \cdots + a_1 s) \,, \tag{17a}$$

$$T_1(s) = a_0 / (a_n s^n + \cdots + a_1 s + a_0) \,. \tag{17b}$$

System type 2, canonical open/closed-loop transfer function, $G_2(s)$ and $T_2(s)$:

$$G_2(s) = (a_1 s + a_0) / (a_n s^n + \cdots + a_2 s^2) \,, \tag{17c}$$

$$T_2(s) = (a_1 s + a_0) / (a_n s^n + \cdots + a_1 s + a_0) \,. \tag{17d}$$

These canonical open/closed-loop transfer functions are uniquely defined by the characteristic polynomial $P(s)$, and they are helpful to visualize the characteristics of $P(s)$. Also break point $\omega_i$ is defined as

$$\omega_i = a_i / a_{i+1} = 1 / \tau_i \,. \tag{18a}$$

The $\omega_i$ is the reciprocal of the equivalent time constant of high order $\tau_i$. The ratio of adjacent break points is equal to the stability index $\gamma_i$.

$$\gamma_i = \omega_i / \omega_{i-1} \tag{18b}$$

Fig. 5 shows an example of Bode diagram of the canonical open-loop transfer function for the system type 1 and 2. The straight-line approximation (asymptotic representation) of Bode diagram used here is somewhat different from the ordinary way. The break points are chosen from the ratio of the coefficients and not from the poles and zeros of the transfer function as in the usual case. However this way is more accurate and the relation with the coefficient diagram is closer.

Thus it becomes clear that the coefficient diagram has a one-to-one correspondence with the straight-line approximation of Bode diagram of its canonical open-loop transfer function.

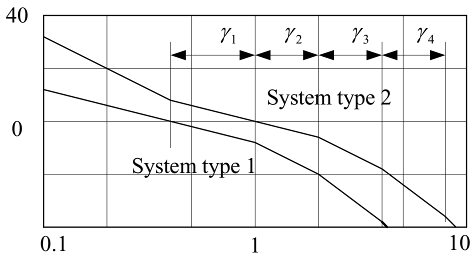

Fig. 5. Canonical open-loop transfer function

### 2.7 Standard form

From number of reasons, which will become clear later, the recommended standard form for CDM is

$$\gamma_{n-1} \sim \gamma_2 = 2, \qquad \gamma_1 = 2.5 . \qquad \text{(19a)}$$

When $a_0$ = 0.4 and $\tau$ = 2.5 are chosen, the characteristic polynomial $P(s)$ is obtained by Eq. (7c) in the following simple form:

$$P(s) = 2^{-\frac{(n-2)(n-1)}{2}} s^n + \cdots + 2^{-10} s^6 + 2^{-6} s^5 + 2^{-3} s^4 + 0.5 s^3 + s^2 + s + 0.4 . \qquad \text{(19b)}$$

The step response of the canonical closed-loop transfer function for the system type 1 and 2 for various orders are given in Fig. 6 and 7. There is virtually no overshoot for the system type 1. There is an overshoot of about 40% for system type 2. This overshoot is necessary, because the integral of the error for the step response must become zero in system type 2. It is also noticed that the responses are about the same irrespective to the order of the system. Because of this nature, the designer can start from a simple controller and move to more complicated one in addition to the previous design. The settling time is about 2.5 ~ 3$\tau$. Many simulation runs show that the standard form has the shortest settling time for the same value of $\tau$.

The pole location is given in Fig. 8. It is found that the three lowest order poles are aligned in a vertical line and the two highest order poles are at the point about 49.5 deg from the negative real axis. The rest of the poles are on or close to the negative real axis. For 4th order, all poles are exactly on the vertical line.

It can be mathematically proven that a 3rd order system with three poles on a vertical line has no overshoot. For $\gamma_2$=2 and $\gamma_1$=2.7, three poles are on a vertical line and overshoot is zero. If $\gamma_1$ = 2.5 as in the standard form, the complex poles are a little bit closer to the imaginary axis with the result of a small overshoot. The choice of $\gamma_1$ = 2.5 instead of 2.7 is made for the reason of simplicity.

In summary, the standard form has the favorable characteristics as listed below.

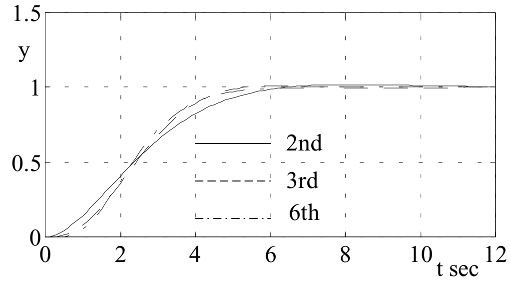

Fig. 6. Step response, system type 1

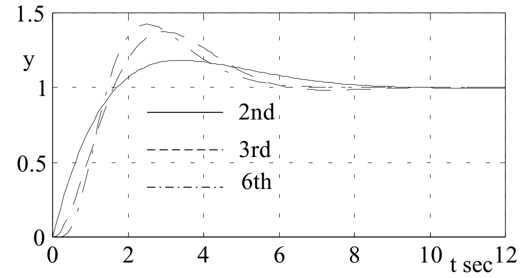

Fig. 7. Step response, system type 2

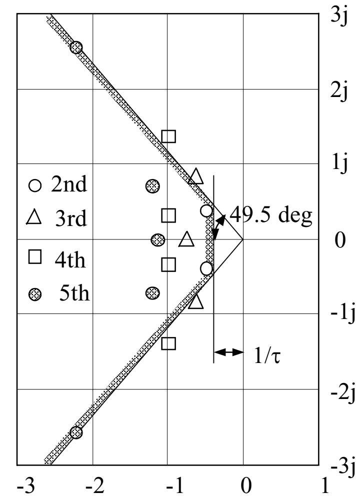

Fig. 8. Pole location

(1) For system type 1, overshoot is almost zero. For system type 2, necessary overshoot of about 40 % is realized.
(2) Among the systems with the same equivalent time constant $\tau$, the standard form has the shortest settling time. The settling time is about 2.5~3$\tau$.
(3) The step responses show almost equal waveforms irrespective to the order of the characteristic polynomials.
(4) The lower order poles are aligned on a vertical line. The higher order poles are located within a sector 49.5 degrees from the negative real axis, and their damping ratio $\zeta$ is larger than 0.65.
(5) The CDM standard form is very easy to remember.

In other words, the standard form seems to posses all the characteristics of "good designs" found from experience, such as no overshoot, short settling time, and pole alignment on a vertical line. For comparison, stability indices $\gamma_i$'s for various standard forms used in the control theory are given in Table 1. It is found that CDM standard is similar to Bessel at the low order, and become similar to binomial at the high order.

Table 1. Comparison of stability index

| Standard forms | Stability index $\gamma_4$ | $\gamma_3$ | $\gamma_2$ | $\gamma_1$ | Standard forms | Stability index $\gamma_4$ | $\gamma_3$ | $\gamma_2$ | $\gamma_1$ |
|---|---|---|---|---|---|---|---|---|---|
| Binomial | | | | 4 | ITAE | | | | 2 |
| | | | 3 | 3 | | | | 1.424 | 2.641 |
| | | 2.667 | 2.25 | 2.667 | | | 1.297 | 2.039 | 2.144 |
| | 2.5 | 2 | 2 | 2.5 | | 1.568 | 1.624 | 1.779 | 2.102 |
| Butter-worth | | | | 2 | Kessler | | | | 2 |
| | | | 2 | 2 | | | | 2 | 2 |
| | | 2 | 1.707 | 2 | | | 2 | 2 | 2 |
| | 2 | 1.618 | 1.618 | 2 | | 2 | 2 | 2 | 2 |
| Bessel | | | | 3 | CDM | | | | 2.5 |
| | | | 2.4 | 2.5 | | | | 2 | 2.5 |
| | | 2.222 | 1.929 | 2.333 | | | 2 | 2 | 2.5 |
| | 2.143 | 1.75 | 1.778 | 2.25 | | 2 | 2 | 2 | 2.5 |

### 2.8 Robustness consideration

Robustness and stability are completely different concepts. Simply stated, stability concerns where the poles are located, and robustness concerns how fast the poles move to the imaginary axis for the variation of parameters. Stability is specified by the stability index $\gamma_i$ of the characteristic polynomial, but robustness is only specified after the open-loop structure is specified. Thus in designing the characteristic polynomial, more consideration is required beyond the choice of $\gamma_i$. The traditional design principle of sticking to the minimum-phase controller wherever possible, with the lowest-possible order, and with the narrowest possible bandwidth is actually found to be a strong guarantee of robustness.

In the actual design, the choice of $\gamma_1 = 2.5,\ \gamma_2 = \gamma_{n-1} = 2$ is strongly recommended due to stability and response requirement, but it is not necessary to make $\gamma_3 \sim \gamma_{n-2}$ equal to 2. The condition can be relaxed as

$$\gamma_i > 1.5\gamma_i^* . \qquad (20)$$

With such freedom, the designer has the freedom of designing the controller together with the characteristic polynomial, and he can integrate robustness in the characteristic polynomial with a small sacrifice of stability and response. From the sufficient condition for stability by Lipatv, stability is guaranteed when all $\gamma_i$'s are larger than 1.5. Lipatov proved in his paper [4] that, if all $\gamma_i$'s are greater than 4, all the roots are negative real. Thus $\gamma_i$'s are usually chosen between 1.5 and 4. Because the essence of the CDM lies in the proper selection of stability indices $\gamma_i$'s, some experiences are required in actual design, as is true in any design effort.

## 3 CONTROLLER DESIGN

The important feature of CDM is the simultaneous design of the characteristic polynomial and controller. By this feature, robustness can be designed into the controller as well as stability and response requirement. In order to make the problem specific, the following example is considered.

$$A_p(s) = d_3 s^3 + d_2 s^2 + d_1 s + d_0 , \qquad (21a)$$

$$B_p(s) = n_2 s^2 + n_1 s + n_0 , \qquad (21b)$$

$$A_c(s) = l_2 s^2 + l_1 s + l_0 , \qquad (21c)$$

$$B_c(s) = k_2 s^2 + k_1 s + k_0 , \qquad (21d)$$

$$B_a(s) = m_2 s^2 + m_1 s + m_0 . \qquad (21e)$$

The characteristic polynomial is given as Eqs. (5)(7c).

$$P(s) = a_n s^n + ... + a_1 s + a_0 = \sum_{i=0}^{n} a_i s^i , \qquad (22a)$$

$$a_i = a_0 \tau^i / \ (\gamma_{i-1}\gamma_{i-2}^2 \cdots \gamma_2^{i-2}\gamma_1^{i-1}) . \qquad (22b)$$

The characteristic polynomial is related to plant and controller polynomials by Eq. (3b), which is commonly called Diophantine equation.

$$P(s) = A_c(s)A_p(s) + B_c(s)B_p(s) \qquad (23)$$

The Diophantine equation can be expanded to a linear relation of coefficients as follows:

$$\begin{bmatrix} d_3 & 0 & 0 & 0 & 0 & 0 \\ d_2 & d_3 & 0 & n_2 & 0 & 0 \\ d_1 & d_2 & d_3 & n_1 & n_2 & 0 \\ d_0 & d_1 & d_2 & n_0 & n_1 & n_2 \\ 0 & d_0 & d_1 & 0 & n_0 & n_1 \\ 0 & 0 & d_0 & 0 & 0 & n_0 \end{bmatrix} \begin{bmatrix} l_2 \\ l_1 \\ l_0 \\ k_2 \\ k_1 \\ k_0 \end{bmatrix} = \begin{bmatrix} a_5 \\ a_4 \\ a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix} . \qquad (24)$$

The matrix of the left side is called Sylvester matrix (SM). If the SM is square and non-singular, the controller parameters can be obtained from the coefficients of the characteristic polynomial. If the plant is controllable and observable, the SM is non-singular. The condition that the SM is square is given as

$$m_c = n_p - 1 , \qquad (25a)$$

where $m_c$ is the order of $B_c(s)$ and $n_p$ is that of $A_p(s)$.

In Eq. (24), the real design freedom is 5, because $a_0$ can be arbitrarily chosen for the same controller. Thus if, by introduction of condition for controller parameter selection, $f_c$ freedom is lost, $m_c$ must increase by that amount. On the other hand, if $f_p$ loss of freedom in characteristic polynomial is acceptable, $m_c$ can be decreased by that amount. Thus Eq. (25a) is modified as

$$m_c = n_p - 1 + f_c - f_p . \qquad (25b)$$

By the proper use of the coefficient diagram, the designer can choose $f_p$ wisely, and the low order controller with good stability, response, and robustness can be realized. This corresponds to selecting a characteristic polynomial best fitted to the plant and control requirements. In this respect, CDM is quite different from the pole assignment.

## 4 Recent Development

Some of the important papers in recent 10 years will be reviewed. Hori used CDM standard form in design of 2-mass control system and gave the PID parameters in explicit form of the plant parameter [2]. Manabe used CDM for the analysis of the attitude control of controlled bias momentum satellite [5], and solution of ACC benchmark problem [6]. Also the tutorial paper for CDM [8] and for Lipatov's sufficient condition for stability [9] were given by Manabe.

The first organized session on CDM was organized by Kim [11]. The second one was organized by Manabe [12]. Kim made comparison of various control approaches [13]. Tesfaye reported the design and experimental result for actual motion control [14]. Photong reported a design example for process control [15]. Pang introduced a CDM-CAD with GUI capability [16]. Hara introduced another CDM-CAD with example [17]. Kim reported an application to MIMO system in steel mill industry [18 ].

Further application to MIMO design were made by Manabe [21] and Hirokawa [20] in missile control. Application to system with dead-time was suggested by Hamamci [19]. Because CDM is closely related with polynomial design approaches using Diophantine equation, efforts were made to exchange ideas with the researchers in that field [22][23].

## 5 Conclusions

In this paper, brief tutorial and short survey of CDM are presented. The CDM is very useful at this stage, but further development is keenly expected especially for good CAD system and application to MIMO system

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