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COEFFICIENT DIAGRAM METHOD AS APPLIED TO THE ATTITUDE CONTROL OF CONTROLLED-BIAS-MOMENTUM SATELLITE

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Abstract. A controller design method, called Coefficient Diagram Method (CDM), is introduced. By this method the designer can design the characteristic polynomial of the closed loop system efficiently taking a good balance of stability, response, and robustness. By CDM, an unified interpretation of various attitude controls of various controlled-bias-momentum satellites is made.

Key Words Control system design; control theory; attitude control; satellite, artificial; satellite control; controllers

1. INTRODUCTION

The purpose of this paper is to give an unified interpretation to various attitude controls of controlled-bias-momentum satellite, where the bias-momentum varies not only in magnitude but also in direction by the use of other wheels.

In order to give an unified interpretation, some kind of fixed standpoint is necessary. The "Coefficient Diagram Method" in control system design is fairly new and not well-known, but its basic philosophy has been used in many fields of industry for more than 30 years with successful application especially in steel mill speed control.

The coefficient diagram is a semi-log diagram where the coefficients of characteristic polynomials are shown in logarithmic scale in the ordinate and the numbers of power corresponding to each coefficient are shown in the abscissa. The degree of convexity is a measure of stability. The general inclination of the curve is a measure of response speed. The variation of the shape of the curve is a measure of robustness. Thus the three major characteristics of control system, namely stability, response, and robustness are shown graphically in a single diagram, enabling the designer to make a balanced judgment in the course of his design.

The power of the coefficient diagram method (CDM) lies in that it generates not only non-minimum phase controllers but also unstable controllers when required. Unstable controllers are shown to be very effective in controlling such unstable plants as inverted pendulums with limited number of sensors (Manabe, 1994). LQG fails to produce a robust controller for plant with flexibility (poles at the vicinity of the imaginary axis) as pointed by various authors (Edmunds, 1983, and Mills, 1992). CDM produces very robust controllers in such cases. The experience show that only well-designed $H\infty$ controller can be equivalent to CDM controllers.

This paper will first explain the historical background which constitutes the basis of CDM. The heart of CDM is the design of the characteristic polynomial, and much attention will be paid on this subject. Finally various control laws for the attitude control of satellites will be compared from the view point of CDM, and their specific features will be explained. A set of control law which will be considered best from the view point of CDM is also suggested.

2. HISTORICAL BACKGROUND

In 1950s, the frequency response method was widely used in control system design. During that period, it was commonly recognized that, for good system design, such criteria as the phase margin or gain margin were not sufficient and the frequency characteristics of the open loop transfer function should have proper shape for a fairly wide frequency range (Tustin, 1958).

Chestnut (1959) pointed out in his celebrated book the importance of the relative location of break points and the change of slope at these break points at the straight line approximation of the Bode diagram (gain only) of the open loop transfer function, and he proposed a design method based on these findings. His proposal was very practical, and has been widely used in industry not only in 1950s but even today.

The rule of thumb, such that the straight line approximation of the gain should intersect the 0 dB line at the slope of -20 dB/dec., the change of slope at each break point should be 20 dB/dec., and the break points (time constant) should be separated at least by factor of two, has been widely used in practical design of simple control systems.

Because the position of these break points and the coefficients of the characteristic polynomial have a very close relation, and the specification of the former is equivalent to that of the latter. Thus these design philosophies constitute the basis of CDM.

Graham (1953) made intensive research to find the relation between the coefficients of characteristic polynomials and the transient response, and proposed standard forms for desirable characteristic polynomials. This is commonly called ITAE (Integral Time Absolute Error) Standard Form. The values of coefficients of this standard form are similar, but a little more oscillatory, compared to the proposed values for CDM.

Around these time, Kessler (1960) made intensive efforts to establish synthesis (design) procedures for multi-loop control systems, and came out with a standard form, commonly called "Kessler Canonical Multi-loop Structure". The proposed system is more stable compared to ITAE standard form, and, for this reason, has been widely used in the steel mill control. It virtually became the industry standard. The CDM is simply the sophistication and generalization of Kessler's work.

Stability of control systems can be analyzed by Routh or Hurwitz criterion utilizing coefficients of characteristic polynomials. However in this way the effect of the variation of coefficients on stability is not clearly seen. Lipatov (1978) proposed sufficient conditions for stability and instability. Because of its simplicity, the relation of stability and instability with respect to the coefficients of the characteristic polynomials becomes very clear. These conditions are integrated to the design procedures of CDM.

In control system design, classical control theory and modern control theory are widely used. But there is other approach called algebraic approach, and Chen (1987) proposed a simple design approach based on this philosophy. His approach is basically sophistication of the pole allocation method for closed loop characteristic polynomials. Some of his idea constitutes basic philosophy of CDM. Although rational functions are commonly used in algebraic approach, only polynomials are used in CDM. In this way, design procedures are much simplified and become more straight forward.

Simply stated, CDM is an algebraic approach using only polynomials, where the coefficient diagram is utilized as a vehicle to collectively express the important features of the system, and an improved version of Kessler's standard form and the stability condition of Lipatov constitute the theoretical basis.

There are three features in CDM which naturally follows from the above mentioned basic structure. First, co-prime factorization forms rather than state space representation or transfer function representation are mainly used to express the mathematical model of the plant and the controller, because these forms are closely related to polynomials and, while very compact in expression, they have the same rigor as the state space representation. Secondly, closed loop design approach rather than open loop design approach is adopted, since in CDM the design of the characteristic polynomial is of utmost importance.

Finally, guided search approach rather than design by specification approach is adopted. The controller is first assumed in the simplest possible form to satisfy the basic specification, and gradually improved to meet full specification. In that process the specification itself is being improved keeping balance between the necessity of the specification and the complexity of required controller. This procedure becomes possible because the characteristics of the system is clearly expressed in the coefficient diagram and the coefficients are explicitly connected to the parameters of the controller.

3. CHARACTERISTIC POLYNOMIAL

In CDM, stability index γ_i, equivalent time constant τ, and stability limit γ_i^* play very important role.

They are defined from the coefficients of the characteristic polynmials as follows;

$$\gamma_i = a_i^2 / (a_{i+1}\, a_{i-1}) \qquad i = 1 \sim n\text{-}1 \tag{1}$$

$$\tau = a_1 / a_0 \tag{2}$$

$$\gamma_i^* = 1/\gamma_{i+1} + 1/\gamma_{i-1} \tag{3}$$

$$\gamma_n\,,\ \gamma_0 = \infty$$

By use of these parameters, the characteristic polynomial is expressed as follows;

$$P(s) = a_0 \left[\left\{ \sum_{i=2}^{n} \left(\prod_{j=1}^{i-1} \frac{1}{\Gamma_j} \right) (\tau s)^i \right\} + \tau s + 1 \right] \tag{4}$$

$$\Gamma_j = \gamma_j\, \gamma_{j-1} \cdots \gamma_2\, \gamma_1 \tag{5}$$

Thus the coefficicnts are

$$a_i = a_0\, \tau^i / (\Gamma_{1-1} \cdots \Gamma_2\, \Gamma_1) \tag{6}$$

$$a_i = a_0\, \tau^i / (\gamma_{1-1}\, \gamma_{i-2}^2 \cdots \gamma_1^{i-1}) \tag{7}$$

Also stability index of higher order is defined as

$$\gamma_{ij} = \frac{a_i^2}{a_{i+j}\; a_{i-j}} = \left[\prod_{k=1}^{j-1} (\gamma_{i+j-k}\;\gamma_{i-j+k})^k \right] \gamma_i^j \tag{8}$$

From these equations, a few important relations among the coefficients and the parameters are obtained as follows;

$$a_{i+1} / a_i = \tau / (\gamma_i \cdots \gamma_2\, \gamma_1) \tag{9}$$

$$a_{i-1}\, a_{j+1} / (a_i\, a_j) = \gamma_{i-1}\, \gamma_{i-2} \cdots \gamma_{j+1} \qquad i > j \tag{10}$$

$$a_i / a_j = [\tau / \Gamma_j]^{i-j} / [\gamma_{i-1}\, \gamma_{i-2}^2 \cdots \gamma_{j+1}^{i-j-1}] \tag{11}$$

$$a_{j+1} / a_j = [\gamma_{i-1}\, \gamma_{i-2}^2 \cdots \gamma_{j+1}^{i-j-1}\, (a_i / a_j)]^{1/(i-j)} \tag{12}$$

When a characteristic polynomial is expressed as

$$P(s) = 0.25\, s^5 + s^4 + 2\, s^3 + 2\, s^2 + s + 0.2 \tag{13}$$

the coefficient diagram is shown as in Fig. 1, where coefficient a_i, stability index γ_i, equivalent time constant τ, and stability limit γ_i^* are shown in one figure. When the curvature of a_i curve becomes large, the system become more stable corresponding to larger γ_i's. When the curve a_i is left-end- down, the equivalent time constant τ is small and the response is fast.

The stability conditions are summalized as follows; By mathematical manipulation of Routh-Hurwitz criterion, the stability condition for 4th and 3rd order system becomes

$$\gamma_i > \gamma_i^* \tag{14}$$

For the system higher than or including 5th degree, the sufficient condition for stability and instabilitity is obtained by Lipatov (1978). The sufficient condition for stability is that either of the following two equations holds.

$$\gamma_i = 1.12\, \gamma_i^* \qquad i = 1 \sim n\text{-}1 \tag{15}$$

$$(\gamma_{i+1}\, \gamma_i)^{0.5} > 1.47 \qquad i = 1 \sim n\text{-}2 \tag{16}$$

The sufficient condition for instability is shown to be

$$(\gamma_{i+1}\, \gamma_i)^{0.5} < 1 \qquad i = 1 \sim n\text{-}2 \tag{17}$$

In CDM, the recommended standard form is

$$\gamma_1 = 2.5, \qquad \gamma_{n-1} = \gamma_{n-2} = \dots = \gamma_2 = 2 \tag{18}$$

The standard form has the favorable characteristics as listed below.

(1) When the order of the numerator polynomial is zero and the order of the characteristic polynomial is equal or higher than four, the system has no overshoot. Only negligible overshoot exists for the second and third order.

(2) Among the system with the same equivalent time constant, the standard form has the shortest settling time. The settling time is about 2.5 ~ 3 τ.

(3) For the same equivalent time constant and the unity numerator, the step responses of the standard form show almost equal wave forms irrespective to the order of the characteristic polynomials.

(4) The characteristic roots of lower order have equal decay characteristics with the almost equal negative real parts and aligned on a vertical line. The characteristic roots for higher order are located within a sector 50 degrees from the negative real axis, and their damping coefficient ζ is larger than 0.64.

(5) The CDM standard form is very easy to remember.

In the actual design, the choice of $\gamma_1 = 2.5$, $\gamma_2 = \gamma_3 = 2$ is strongly recommended, but it is not necessary to make $\gamma_4 \sim \gamma_{n-1}$ equal to 2. The condition can be relaxed as

$$\gamma_i > 1.5\, \gamma_i^* \tag{19}$$

In such case, the roots of the higher order become constant decay and the damping coefficient becomes smaller. The robustness decreases slightly, but it is largely offset by narrower bandwidth, increase of design flexibility, and lower order controller.

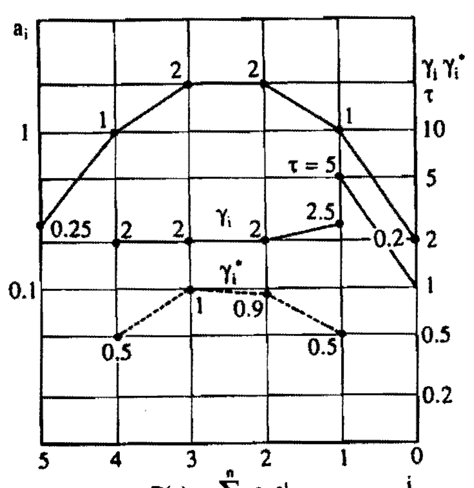

Fig.1. Coefficient diagram

Because the essence of the CDM lies in the proper selection of stability indices γ_i's, some experiences are required in actual design, as is true in any design effort.

4. COMPARISON OF CONTROL LAWS

Control laws previously suggested. Various control laws are suggested for the attitude control of the controlled-bias-momentum satellite (Manabe, 1981). In a simplified form normalized by the nutation frequency, the control system is shown as in Fig. 2. The definition of notations are as follows;

ϕ_r = the roll angle reference in radian

ϕ = the roll angle in radian

$u = h_z / h_m$ = the ratio of the momentum of the yaw wheel to the momentum of the pitch wheel (minus pitch direction being positive)

d = momentum disturbance due to the body residual momentum (this term is set to zero for simplicity)

The open loop transfer functions G(s), stability indices γ_i's, and closed loop poles s_i's are shown in Table 1. The systems from 1 to 4 are proposed in literatures. The systems 5 and 6 are loosely named as Zero PID control. Zero PID control is a special PID control, where the proportional gain is almost zero.

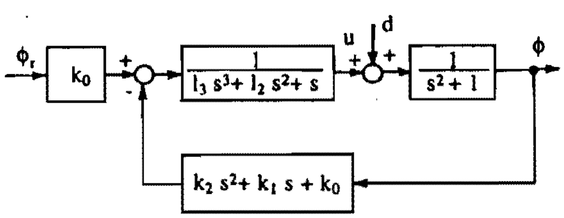

Fig. 2. Attitude control system

Control laws derived by CDM. By use of CDM, the attitude controllers, with different features, for the controlled-bias -momentum satellites can be derived in a systematic manner. From Fig. 2, the characteristic polynomial is obtained as

$$P(s) = l_3 s^5 + l_2 s^4 + (l_3 + 1) s^3 + (l_2 + k_2) s^2 + (1 + k_1) s + k_0 \tag{20}$$

The absence of the l_0 term in Fig. 2 is due to the requirement that the roll angle error must be zero for the presence of momentum disturbance.

In Eq. (20), the parameter k_1 plays a very important role. From Eqs. (2) and (7),

$$a_1 = 1 + k_1 = a_0 \tau \tag{21}$$

$$a_3 = 1 + l_3 = a_0 \tau^3 / (\gamma_2 \gamma_1^2) \tag{22}$$

Thus for $\gamma_2 = 2$ and $\gamma_1 = 2.5$, by division of a_3 by a_1

$$\tau = 3.536 [(1 + l_3) / (1 + k_1)]^{0.5} \tag{23}$$

The case where $l_3 = 0$ and $k_1 = 0$ is chosen as the standard case. The equivalent time constant τ is 3.536 for this case. If a system of faster response, having the same stability, is required, k_1 must be increased while l_3 is kept to zero in order to make τ small. If the compensator of order n-m is defined as a compensator whose denominator is order n and numerator order m, the compensator for this case is order 2-2.

If a system of slower response is required, k_1 must be decreased to the negative direction, maybe down to - 0.5. Further decrease is not advisable because small variation of k_1 may cause a large percentage change of a_1. In this case, increase of l_3 is also helpful, as is clear from Eq. (23). Thus the compensator for this case is order 3-2.

In order to realize a larger l_3, the left most stability index γ_4 may be decreased to 1 from 2. This selection doubles the value of l_3. When k_1 and γ_i's are given, l_3 can be obtained from the following relation derived from Eq. (8).

$$a_3^2 / (a_5 a_1) = (1 + l_3)^2 / [l_3 (1 + k_1)] = \gamma_4 \gamma_3^2 \gamma_2 \tag{24}$$

Table 3 shows the control laws derived by CDM. The k_1's are chosen to be 0, 1, -0.3333, and -0.5. They are named as "standard", "high gain", "medium-low gain", and "low gain". The former two systems are the 4th order and γ_i = [2 2 2.5] , while the latter two are the 5th order and γ_i = [1 2 2 2.5] . Examination of closed loop poles will reveal that, for the same γ_i's, the configuration of the poles does not change; only the distances to the origin change in the same proportion.

Although all systems have sufficient stability and practically sufficient robustness, the higher the gain k_1 is, the system is the more robust. However, the higher gain system requires wider bandwidth, and some caution is necessary for the effect of noise. Thus the system designers are requested to select proper k_1 to meet to their specific requirements of robustness and bandwidth.

Comparison of control laws. Many interesting observations can be made by the comparison of Tables 1 and 2. First, striking similarity between Terasaki's control law and CDM low gain is noticed. Although these systems enjoy the narrowest bandwidth, these are least robust and become unstable for the loop gain increase by factor of 2. They are stable for the gain decrease. In essence, CDM low gain is nothing but a sophistication of Terasaki's control law, in the sense that shorter settling time and smoother response are realized even with narrower bandwidth and with the same degree of robustness, as shown in Figs. 3 and 4.

The second observation is that CDM medium-low gain looks to be an improvement over Dahl and Tsuchiya with respect to settling time, smoothness of response, and robustness.

The third observation is that CDM standard is similar to ID control if the gain k_0 is increased by 1.414 times. If such increase of gain is made, the characteristic polynomials become almost the same. In other words, ID Control can be interpreted as CDM standard, where the gain k_0 is reduced to 70 percent in order to attain better robustness at the increase of loop gain. Although the response of ID Control is more sluggish, the degradation at gain increase by factor of 2 is very small. The comparison of the response is shown in Fig. 5 and 6. Although ID control is a simplification of skewed lead wheel, it is more close to CDM Standard.

Some calculation reveals that the closed loop pole configuration of CDM medium-low and low gain can not be attained in LQG design, if the positive definiteness, or semi-definiteness for the weight matrix of states is to be observed. If LQG design is to be used to produce the same pole configuration, one of the following three methods (These are equivalent.) may be used.
(1) Choose an indefinite weight matrix for states.
(2) Use proper cross product weight of states and the plant input.
(3) Put a proper positive pre-feedback to the system and apply the standard LQG.

In any case, the design becomes very complicated. Experience shows that it is always possible to find better controllers by CDM, because of the limitations imposed on the standard LQG, namely, the positive definiteness or semi-definiteness of the weight matrix of states.

Table 1 Comparison of control laws

1. Terasaki

$$G(s) = \frac{-0.5s + 0.09375}{(0.4444s^3+1.3333s^2+s)(s^2+1)}$$

$\gamma_i = [2.769\ \ 1.174\ \ 2.461\ \ 2]$, $\tau = 5.333$

$s_i = -0.2583 \pm j\,0.2326$, -1.9970, $-0.2434 \pm j\,0.9029$

2. Lebsock

$$G(s) = \frac{-0.4694s + 0.0875}{(1.429s^2+s)(s^2+1)}$$

$\gamma_i = [0.490\ \ 3.848\ \ 2.252]$, $\tau = 6.065$

$s_i = -0.2171 \pm j\,0.1690$, $-0.1328 \pm j\,0.8896$

3. Dahl

$$G(s) = \frac{0.312s^2 - 0.416s + 0.104}{(0.667s^2+s)(s^2+1)}$$

$\gamma_i = [1.531\ \ 1.641\ \ 3.350]$, $\tau = 5.615$

$s_i = -0.2771$, -0.7307, $-0.2457 \pm j\,0.8424$

4. Tsuchiya

$$G(s) = \frac{0.4317s^2 - 0.3796s + 0.1472}{(0.118s^3+0.605s^2+s)(s^2+1)}$$

$\gamma_i = [2.775\ \ 1.993\ \ 1.550\ \ 2.522]$, $\tau = 4.215$

$s_i = -0.4424$, -1.8608, -2.1535, $-0.3352 \pm j\,0.7689$

5. Skewed lead wheel

$$G(s) = \frac{0.3636s^3 + s^2 + 0.07272s + 0.2}{(0.3636s^2+s)(s^2+1)}$$

$\gamma_i = [3.750\ \ 1.271\ \ 4.219]$, $\tau = 5.364$

$s_i = -0.2455$, -2.7503, $-0.3773 \pm j\,0.8200$

6. ID control

$$G(s) = \frac{s^2 + 0.07272s + 0.2}{(0.3636s^2+s)(s^2+1)}$$

$\gamma_i = [2.017\ \ 1.733\ \ 4.219]$, $\tau = 5.364$

$s_i = -0.2551$, -1.371, $-0.5621 \pm j\,1.1210$

Table 2 Control law derived by CDM

1. CDM standard

$$G(s) = \frac{1.0606s^2 + 0.2828}{(0.3536s^2+s)(s^2+1)}$$

$\gamma_i = [2\ \ 2\ \ 2.5]$, $\tau = 3.536$

$s_i = -0.7070 \pm j\,0.2294$, $-0.7070 \pm j\,0.9734$

2. CDM high gain

$$G(s) = \frac{1.75s^2 + s + 0.8}{(0.25s^2+s)(s^2+1)}$$

$\gamma_i = [2\ \ 2\ \ 2.5]$, $\tau = 2.5$

$s_i = -1.000 \pm j\,0.3249$, $-1.000 \pm j\,1.3764$

3. CDM medium-low gain

$$G(s) = \frac{0.6666s^2 - 0.3333s + 1.3333}{(0.3333s^3+0.6667s^2+s)(s^2+1)}$$

$\gamma_i = [1\ \ 2\ \ 2\ \ 2.5]$, $\tau = 5$

$s_i = -0.4691 \pm j\,0.3624$, -0.4797, $-0.2911 \pm j\,1.5128$

4. CDM low gain

$$G(s) = \frac{-0.5s + 0.07071}{(s^3+1.4142s^2+s)(s^2+1)}$$

$\gamma_i = [1\ \ 2\ \ 2\ \ 2.5]$, $\tau = 7.071$

$s_i = -0.3317 \pm j\,0.2562$, -0.3392, $-0.2058 \pm j\,1.0697$

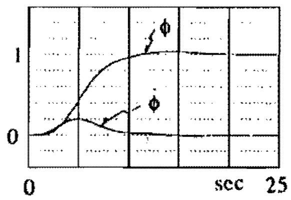

Fig. 3 Terasaki

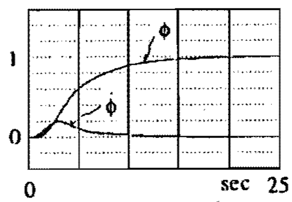

Fig. 5. ID control

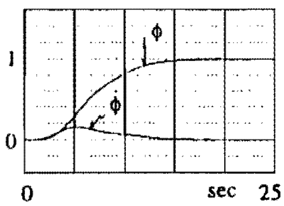

Fig. 4 CDM low gain

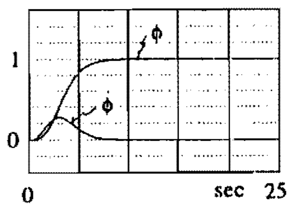

Fig. 6. CDM standard

5. CONCLUSIONS

The major results of this paper are as follows;
(1) The historical background and the outline of the coefficient diagram method (CDM) are briefly explained. The CDM is an algebraic approach using only polynomials, where the coefficient diagram is utilized as a vehicle to collectively express the important features of the system, and an improved version or Kessler's standard form and the stability condition of Lipatov constitute the theoretical basis.

(2) Systematic design is made for the attitude control of controlled-bias-momentum satellite by CDM, where a single gain parameter k_1 is varied as a parameter. It is found that the Terasaki's control law corresponds to CDM Low Gain case and ID Control law to CDM Standard case.

It is also very impressive to note that Terasaki came up to such a remarkable control law at an early stage of the space development and at the time when analysis tools were not widely available as are today. The author expresses his sincere gratitude to all space pioneers like him. Without their toil and insight, the present work would not have been possible

6. REFERENCES

Anzai, T.,K. Okada and others (1989). Attitude and orbit control subsystem for ETS-V and its flight Experiences. pp. 89-94, XIth IFAC Symposium on Automatic Control in Aerospace, Tsukuba, Japan, July 17-21, 1989

Chen, C. T. (1987). Introduction to the linear algebraic method for control system design. *IEEE Contr. Syst. Mag.*, **7**, **5**, 36-42

Chestnut, H. and R. W. Mayer (1959). *Servomechanism and regulating system design.* vol. 1, chapter 14, John Wiley

Dahl, P.R. (1971). A twin wheel bias/reaction jet spacecraft control system. AIAA Paper 71-951, Hofstra University, Hemstead, New York, august 16-18, 1971.

Edmunds, R. S. and D. L. Mingori (1983). Robust control system design techniques for large flexible space structures having non-colocated sensors and actuators. AIAA Paper 83-2294, AIAA Guidance and Control Conference, Aug. 15-17, 1983

Graham, D. and R. C. Lathrop (1953). The synthesis of "optimum" transient response: criteria and standard forms. *AIAA Transactions*, **72**, pt. II, 273-288

Kessler, C. (1960). Ein Beitrag zur Theorie mehrschleifiger Regelungen. *Regelungstechnik*, **8**, **8**, 261-166.

Lebsock, K.A. (1980). High pointing accuracy with a momentum bias attitude control system. *AIAA J. Guidance and Control*, **3**, **3**, 195-202.

Lipatov, A. V. and N. I. Sokolov (1978). Some sufficient conditions for stability and instability of continuous linear stationary systems. translated from *Automatika i Telemekhanika*, no. 9, pp. 30-37, 1978; in *Automat. Remote Contr.*, **39**, 1285-1291, 1979

Manabe, S., K. Tsuchiya and M. Inoue (1981). Zero PID control for bias momentum satellites. Paper 76.4, IFAC 8th World Congress, Kyoto, Japan, Aug. 24-28, 1981

Manabe, S. (1994). A low-cost inverted pendulum system for control system education. to be published in the 3rd IFAC symposium on advances in control education, Tokyo, August 1-2, 1994

Matsueda, T., K. Okada and others (1989). Attitude and orbit control subsystem for MOS-1. pp. 83-88, XIth IFAC Symposium on Automatic Control in Aerospace, Tsukuba, Japan, July 17-21, 1989

Mills, R. A. and A. E. Bryson (1992). Parameter-robust control design using a minimax method. *AIAA Journal of Guidance, Control, and Dynamics*, **15**, **5**, 1068-1075

Terasaki, R.M. (1967). Dual reaction wheel control of spacecraft pointing. Presented at symposium on Attitude Stabilization and Control of Dual-Spin Spacecraft, The Aerospace Corporation, August 1967

Tsuchiya, K. and M. Inoue (1980). Advanced reaction wheel controller for space attitude control. IAF-80- E237, XXXI Congress International Aeronautical Federation, Sept. 21-28, 1980

Tustin, A. and others (1958). Position control of massive objects. *Proc. IEE Part C Supplement No. 1*, **105**, 1-57