DIOPHANTINE EQUATIONS IN COEFFICIENT DIAGRAM METHOD

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Abstract: The Coefficient Diagram Method (CDM) is applied to the design of the normal acceleration control of dual-control-surface missile. It is shown that MIMO design problem (matrix Diophantine equation) can be decomposed into series of SIMO problems (scalar Diophantine equation). Feedback controller is designed step by step with CDM. The extra freedom in design, typical in MIMO system, is used freely to design feed-forward controller, whereby various system characteristics can be attained.

Keywords: Control system design, Missiles, MIMO, Polynomials, Algebraic approaches.

1. INTRODUCTION

There are two distinct problems, when algebraic approach is used for control system design, namely definition of the Diophantine equation (DEQ) and the solution of the equation. The former corresponds to the total design of the control system and the latter to the design of the controller for the system.

However, in actual practice, these two problems are mixed. The designer starts from the incomplete specification of the closed-loop performance and vague definition of controller limitation, and by improving step by step finally ends up to the complete definition of the control system and the controller.

The purpose of this paper is to show, by an example, the definition and solution cycle of the DEQ. The problem is the normal acceleration control of a dual-control-surface missile (Ochi, 1997), and it is solved by the coefficient diagram method (CDM), where the MIMO problem (matrix DEQ) is decomposed into series of SIMO problems (scalar DEQ), and each SIMO problem is solved by standard CDM procedure.

This paper is organized as follows. In Section 2, the basics of CDM are explained. In Section 3, the mathematical model of a dual-control-surface missile is presented. In Section 4, the basic control structure is determined based on coefficient diagram analysis. In Section 5, a feedback controller is designed as a SIMO problem by CDM. In Section 6, various feed-forward controllers are suggested. In Section 7, the problems encountered in MIMO design are discussed.

2. BASICS OF CDM

2.1 Basic Philosophy of CDM

The CDM is an algebraic control design approach with the following five features (Manabe, 1998b).

(1) Polynomials and polynomial matrices are used for system representation.
(2) Characteristic polynomial and controller are simultaneously designed.
(3) Coefficient diagram is effectively utilized.
(4) The sufficient condition for stability by Lipatov (1978) constitutes the theoretical basis of CDM (Manabe, 1999).
(5) Kessler (1960) standard form is improved and used as the standard form of CDM.

CDM design is based on the stability index and equivalent time constant as defined later. Thus for the specified settling time, a controller of the lowest order with the narrowest bandwidth and of no-overshoot can be easily designed. CDM can be considered as “Generalized PID”, because the controller can be more complex than PID, and more reliable parameter selection rules are provided. Also CDM can be considered as “Improved LQG”, because the order of controller is smaller and weight selection rules are also given (Manabe, 1998a).

2.2 Mathematical Model

The standard block diagram of the CDM design is shown in Fig. 1. This diagram is valid to SISO, SIMO, and MIMO. In SISO, the variables and components are all scalars but in MIMO they are vectors and matrices of proper dimension. The plant equation is given as

$$A_p(s)x = u + d \qquad (1a)$$

$$y = B_p(s)x\,, \qquad (1b)$$

where u, y, and d are input, output, and disturbance. The symbol x is called the basic state variable. $A_p(s)$ and $B_p(s)$ are the denominator and numerator polynomial matrix of the plant. It will be easily seen that this expression has a direct correspondence with the control canonical form of the state-space expression, and x corresponds to the state variable of the lowest order. All the other states are expressed as the derivatives of x of high order.

Controller equation is given as

$$A_c(s)u = B_a(s)y_r - B_c(s)(y+n)\,, \qquad (2)$$

where y_r and n are reference input and noise on the output. $A_c(s)$ is the denominator polynomial matrix of the controller. $B_a(s)$ and $B_c(s)$ are called the reference and feedback numerator polynomial matrix of the controller. Because the controller transfer function has two numerators, it is called two-degree-of-freedom system. This expression corresponds to the observer canonical form of the state-space expression.

Elimination of y and u from Eq. (2) by Eqs. (1a, b) gives

$$A(s)x = B_a(s)y_r + A_c(s)d - B_c(s)n\,, \qquad (3a)$$

where $A(s)$ is the closed-loop system polynomial matrix and given as

$$A(s) = A_c(s)A_p(s) + B_c(s)B_p(s)\,. \qquad (3b)$$

The characteristic polynomial $P(s)$ is given as,

$$P(s) = \det A(s)\,. \qquad (3c)$$

The input output relation is given as

$$\begin{bmatrix} x \\ y \\ u \end{bmatrix} = \frac{1}{P(s)} \begin{bmatrix} I \\ B_p(s) \\ A_p(s) \end{bmatrix} \operatorname{adj} A(s)$$

$$[B_a(s)y_r + A_c(s)d - B_c(s)n] - \begin{bmatrix} 0 \\ 0 \\ d \end{bmatrix}. \qquad (4)$$

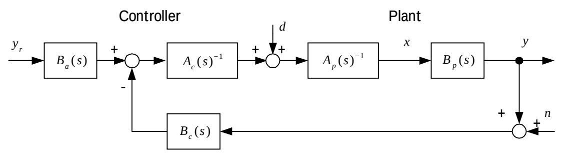


Fig. 1. CDM standard block diagram

2.3 Basic Relations

Some mathematical relations extensively used in CDM will be introduced hereafter. These relations will be freely used in later sections. The characteristic polynomial $P(s)$ is given in the following form.

$$P(s) = a_n s^n + \cdots + a_1 s + a_0 = \sum_{i=0}^{n} a_i s^i \qquad (5)$$

The stability index γ_i, the equivalent time constant τ, and the stability limit γ_i^* are defined as follows.

$$\gamma_i = a_i^2 / (a_{i+1} a_{i-1})\,, \quad i = 1 \sim n-1\,, \qquad (6a)$$

$$\tau = a_1 / a_0\,, \qquad (6b)$$

$$\gamma_i^* = 1/\gamma_{i+1} + 1/\gamma_{i-1}\,, \qquad (6c)$$

γ_n and γ_0 are defined as ∞.

The equivalent time constant of the i-th order τ_i is defined in the similar manner as τ.

$$\tau_i = a_{i+1} / a_i \qquad (7)$$

By Eqs. (6a) and (7),

$$\tau_i / \tau_{i-1} = (a_{i+1}/a_i)(a_i/a_{i-1}) = 1/\gamma_i\,, \qquad (8a)$$

$$\tau_i = \tau_{i-1}/\gamma_i = \tau/(\gamma_i \cdots \gamma_2\,\gamma_1)\,. \qquad (8b)$$

By repeated use of Eqs. (7) a_i is expressed by τ_i and a_0.

$$a_i = \tau_{i-1} \cdots \tau_1 \tau a_0 \qquad (9a)$$

By Eq. (8b), this reduces to

$$a_i = a_0\, \tau^i / (\gamma_{i-1}\gamma_{i-2}^2 \cdots \gamma_2^{i-2}\,\gamma_1^{i-1})\,, \quad i \ge 2\,. \qquad (9b)$$

Then characteristic polynomial is expressed by a_0, τ, and γ_i as follows.

$$P(s) = a_0 \Big[\Big\{\sum_{i=2}^{n} \Big(\prod_{j=1}^{i-1} 1/\gamma_{i-j}^{j}\Big)(\tau s)^i\Big\} + \tau s + 1\Big] \qquad (9c)$$

The sufficient condition for stability and instability constitutes the theoretical basis of CDM. It states as follows.

"The system of any order is stable, if all the partial 4th order polynomials are stable with the margin of 1.12. The system is unstable if some partial 3rd order polynomial is unstable."

Thus the sufficient condition for stability is given as

$$a_i > 1.12\Big[\frac{a_{i-1}}{a_{i+1}} a_{i+2} + \frac{a_{i+1}}{a_{i-1}} a_{i-2}\Big]\,, \qquad (10a)$$

$$\gamma_i > 1.12\gamma_i^*\,, \quad \textit{for all } i = 2 \sim n-2\,. \qquad (10b)$$

The sufficient condition for instability is given as

$$a_{i+1} a_i \le a_{i+2} a_{i-1}\,, \qquad (11a)$$

$$\gamma_{i+1}\gamma_i \le 1\,, \quad \textit{for some } i = 1 \sim n-2\,. \qquad (11b)$$

These conditions are graphically expressed in the coefficient diagram, and the designer can intuitively assess the stability of the system. Fig. 2a is a 4th-order example. Point A is obtained by drawing a line from a_4 in parallel with line $a_3 a_1$. Similarly point B is obtained by drawing a line from a_0 in parallel with line $a_3 a_1$. The stability condition is a_2 > (A + B). The other condition is $\gamma_2 > \gamma_2^*$. Fig. 2b is a 3rd-order example. Point A is $(a_2 a_1)^{0.5}$ and point B is $(a_3 a_0)^{0.5}$. Thus if A is below B, the system is unstable. Point C is $(\gamma_2\gamma_1)^{0.5}$. If it is below 1, the system is unstable.

In CDM, the following stability indices are recommended.

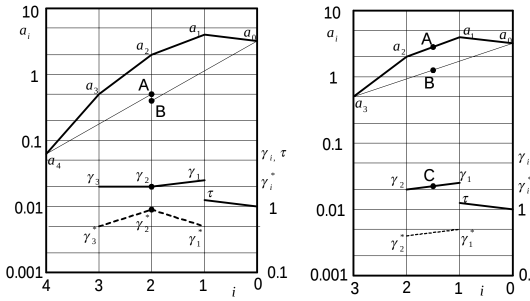
$$\gamma_{n-1} = \ldots = \gamma_3 = \gamma_2 = 2, \quad \gamma_1 = 2.5 \qquad (12a)$$

For more relaxed form, with very small sacrifice of stability,

$$\gamma_i > 1.5\gamma_i^*, \quad i = n-1 \sim 4$$

$$\gamma_3 = \gamma_2 = 2, \quad \gamma_1 = 2.5 . \qquad (12b)$$

In these cases, the step response of Eq. (4) has no overshoot, and the settling time is about 2.5~3τ .



$P(s) = 0.0625s^4 + 0.5s^3 + 2s^2 + 4s + 3.2$ $\qquad$ $P(s) = 0.5s^3 + 2s^2 + 4s + 3.2$

Fig. 2a. 4th order $\qquad$ b. 3rd order

3. MATHEMATICAL MODEL OF DUAL-CONTROL-SURFACE MISSILE

The dual-control-surface missile is sown in Fig. 3. This example is selected from an open literature (Ochi, 1997). This missile has length of 5 m. The operating condition is 10,000 m in altitude and 3 M (900m/sec) in speed. The required normal acceleration is 25 g or about 250 m/sec^2. The state equation is given as

$$\begin{bmatrix} \dot{w} \\ \dot{q} \\ \dot{\delta}_f \\ \dot{\delta}_r \\ a \\ \alpha \end{bmatrix} = \begin{bmatrix} -0.746 & 900 & -240 & -240 & 0 & 0 \\ 0.0532 & -0.572 & 155 & -191 & 0 & 0 \\ 0 & 0 & -100 & 0 & 100 & 0 \\ 0 & 0 & 0 & -100 & 0 & 100 \\ 0.746 & 0.0991 & 240 & 240 & 0 & 0 \\ 1/900 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} w \\ q \\ \delta_f \\ \delta_r \\ u_f \\ u_r \end{bmatrix}, \qquad (13)$$

where w is the downward speed in m/sec; q is the pitch rate in rad/sec; δ_f and δ_r are the deflection of the front and rear control surfaces in rad; a is the normal acceleration in m/sec^2; α is the angle of attack in rad and defined as w/U, where U is the forward speed. The actuator dynamics is represented by a time lag of 0.01 sec, and control inputs are u_f and u_r.

The simplified model for control design is deduced from this model. First, the actuator dynamics is neglected.

$$\delta_f = u_f , \quad \delta_r = u_r \qquad (14a)$$

Then virtual input u_a is defined as

$$u_a = u_f + u_r . \qquad (14b)$$

Then

$$u_f = u_a - u_r . \qquad (14c)$$

By Eqs. (14a, b, c), the design model is derived as

$$\begin{bmatrix} \dot{w} \\ \dot{q} \\ a \\ q \\ a \end{bmatrix} = \begin{bmatrix} -0.746 & 900 & -240 & 0 \\ 0.0532 & -0.572 & 155 & -346 \\ 0.746 & 0.0991 & 240 & 0 \\ 0 & 1 & 0 & 0 \\ 1/900 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} w \\ q \\ u_a \\ u_r \end{bmatrix} . \qquad (15)$$

For easier manipulation, MATLAB style expression of vector is employed, where a column vector [1 3 6]^T is represented as [1; 3; 6]. Then the polynomial matrix expression is given as

$$A_m(s)[w;q] = B_u[u_a;u_r] ,$$

$$[a;q;\alpha] = B_m(s)[w;q] ,$$

$$A_m(s) = \begin{bmatrix} s+0.746 & -900 \\ -0.0532 & s+0.572 \end{bmatrix} ,$$

$$B_m(s) = \begin{bmatrix} -s & 900.0991 \\ 0 & 1 \\ 1/900 & 0 \end{bmatrix} ,$$

$$B_u = \begin{bmatrix} -240 & 0 ; & 155 & -346 \end{bmatrix} . \qquad (16)$$

CDM-standard polynomial matrix model is obtained as

$$A_p(s)[w_1;q_1] = [u_a;u_r] ,$$

$$[a;q;\alpha] = B_p(s)[w_1;q_1] ,$$

$$[w_1;q_1] = B_{u1}^{-1}[w;q] , \quad B_{u1} = [-240\ \ 0; 0\ \ -346] ,$$

$$w_1 = -0.0041667w = -3.75\alpha ,$$

$$q_1 = -0.0028902q ,$$

$$A_p(s) = B_u^{-1}A_m(s)B_{u1} ,$$

$$= \begin{bmatrix} s+0.746 & -1297.5 \\ 0.44798s + 0.29729 & s-580.68 \end{bmatrix} ,$$

$$B_p(s) = B_m(s)B_{u1} .$$

$$= \begin{bmatrix} 240s & -311430 \\ 0 & -346 \\ -240/900 & 0 \end{bmatrix} . \qquad (17)$$

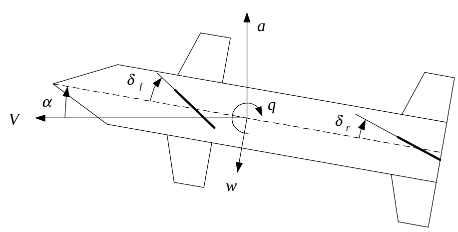


Fig. 3. Dual-control-surface missile

4. CONTROL STRUCTURE DESIGN

The purpose of control is to make the normal acceleration a to follow the command a_r . Thus the number of main output is only one, but there are 2 inputs and 3 outputs, and excessive freedom is left to designer. In order to determine the basic control structure, the 6 input-output relations are derived as

$$[a;q;\alpha]=[1/A_{pd}(s)]B_p(s)\,\mathrm{adj}\,A_p(s)[u_a;u_r],$$

$$A_{pd}(s)=\det A_p(s)=s^2+1.3180s-47.453,$$

$$B_p(s)\,\mathrm{adj}\,A_p(s)=[baa\ \ bar;bra\ \ bqr;b\alpha a\ \ b\alpha r]$$

$$baa=240s^2+152.64s+92586,$$

$$bar=-34.289s-232330,$$

$$bqa=155s+102.86,\quad bqr=-346s-258.12,$$

$$b\alpha a=-0.266667s+154.85,\quad b\alpha r=-346. \qquad (18)$$

These result are obtained by Polynomial Toolbox (Kwakernaak, 2000); These polynomials are shown in the coefficient diagram of Fig. 4, where the coefficient of s^i is multiplied by 10^i for convenience. The minus value is indicated by (-).

For denominator $A_{pd}(s)$, the 1st order coefficient is too small and the 0th order coefficient is negative. These two coefficients must be modified by the feedback. The 1st order coefficient can be modified by proper feedback involving bqa and bqr. Because these two signals are almost alike, either of them can be used. The 0-th order coefficient can be modified by bar, $b\alpha a$, and $b\alpha r$. However baa can not be used, because it affects both 2nd and 0-th order.

From this analysis, the most reasonable feedback control is to control u_r by a and q, while keeping u_a equal to zero.

The signal a and α are similar in nature. Since a is necessary for control anyway, there is no need for α in feedback control. As explained later, α can be controlled effectively by feed-forward control.

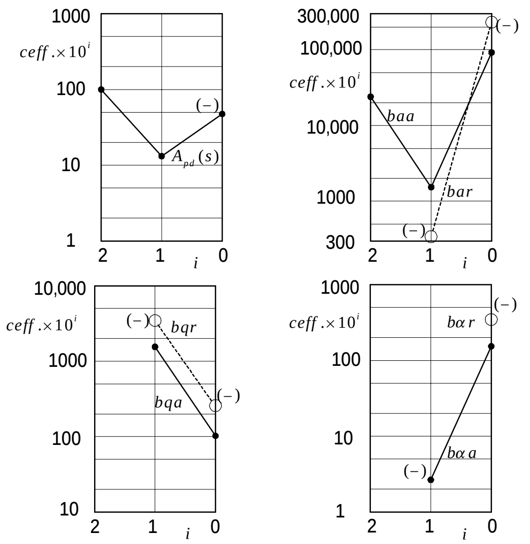


Fig. 4. Denominator and numerator polynomial

5. FEEDBACK CONTROL DESIGN

By making u_a = 0, the plant becomes a SIMO system and expressed as

$$[a;q;\alpha]=[1/A_{pd}(s)][bar;bqr;b\alpha r]\,u_r. \qquad (19a)$$

The natural choice of controller is a PID controller with the feedback of q and a such as

$$su_r=k_0a_r-[k_2sq+(k_1s+k_0)a]. \qquad (19b)$$

The block diagram is shown in Fig. 5. From Eqs. (3b, c), the characteristic polynomial $P(s)$ becomes

$$P(s)=sA_{pd}(s)+k_2s\,(bqr)+(k_1s+k_0)\,(bar)$$

$$=s^3+a_2s^2+a_1s+a_0. \qquad (19c)$$

This is a 3rd order system, and γ_2 and γ_1 are chosen as the standard value.

$$\gamma_2=2,\quad \gamma_1=2.5 \qquad (20a)$$

Because the neglected actuator dynamics has time constant of 0.01 sec, the good value of τ_2 is 0.02 sec, twice as large. By Eq. (8b),

$$\tau=0.1. \qquad (20b)$$

Then by Eq. (9b),

$$a_2=50,\quad a_1=1250,\quad a_0=12500 \qquad (20c)$$

By solving Diophantine equation, Eq. (19c), controller parameters are obtained as

$$k_2=-0.140162,\quad k_1=-0.0054209,$$

$$k_0=-0.053803. \qquad (20d)$$

The solution is easily obtained by CDM CAD (MSS, 2000). The coefficient diagram is shown in Fig. 6.

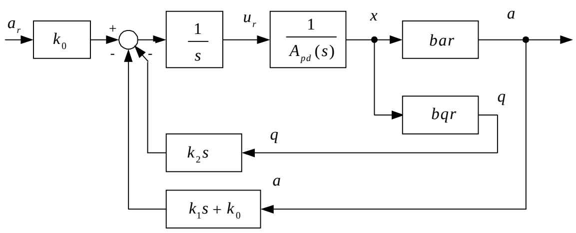


Fig. 5. SIMO block diagram

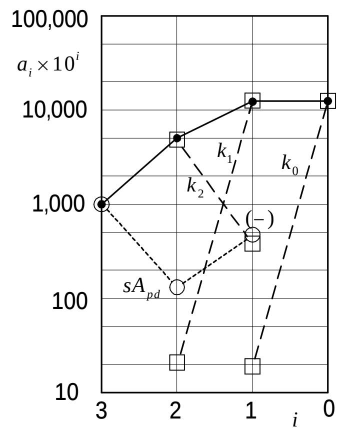


Fig. 6. Coefficient diagram of SIMO design

6. FEED-FORWARD CONTROL DESIGN

The closed-loop polynomial matrix equation, such as Eqs. (1b) and (3a), can be obtained from Eqs. (17) and (19b) as

$$A(s)[\alpha;q]=B_a(s)[u_a;a_r],$$

$$[a;q;\alpha]=B_p(s)[\alpha;q],$$

$$A(s) = A_c(s)A_p(s) + B_c(s)B_p(s) \,,$$
$$A_c(s) = [1\,0; 0\ s] \,,$$
$$B_c(s) = \begin{bmatrix}0 & 0 & 0; & k_1 s + k_0 & k_2 s & 0\end{bmatrix} \,,$$
$$B_a(s) = [1\ 0; 0\ k_0] \,. \qquad (21)$$

Then as Eqs. (17) and (18), six input-output relations are obtained as

$$[a; q; \alpha] = [1/P(s)]\, \mathrm{adj}\, A(s)\ B_a(s)[u_a; a_r] \,,$$
$$P(s) = \det A(s) = s^3 + 50s^2 + 1250s + 12500 \,,$$
$$\mathrm{adj}\, A(s)\ B_a(s) = [b_{11}\ b_{12}; b_{21}\ b_{22}; b_{31}\ b_{32}] \,,$$
$$b_{11} = 240s^3 + 11792s^2 + 92586s \,,$$
$$b_{12} = 1.8448s + 12500 \,,$$
$$b_{21} = -295.15s^2 - 4364.9s \,,$$
$$b_{22} = 18.616s + 13.887 \,,$$
$$b_{13} = -0.266667s^2 - 308.29s - 4468.3 \,,$$
$$b_{32} = 18.616 \,. \qquad (22)$$

When only a is considered as output, relation becomes

$$a = [1/P(s)](b_{11}\, u_a + b_{12}\, a_r) \,. \qquad (23a)$$

The polynomial b_{11} is expressed as

$$b_{11} = 240(s^3 + 49.132s^2 + 385.77s) \,. \qquad (23b)$$

The 3rd and 2nd order coefficients are very similar to those of $P(s)$. The polynomial b_{12} is almost equal to $P(0)$. Now a feed-forward controller is assumed as

$$[u_a; a_r] = [1\ \ 0; 240(T_1 s + 1)\ \ 1][a_f\ / 240; e_1 a_r^*] \,. \qquad (23c)$$

Eq. (23a) becomes

$$a = [1/P(s)][\{b_{11}/240 + (T_1 s + 1)b_{12}\}a_f + b_{12}\, e_1 a_r^*] \,. \qquad (23d)$$

When T_1 is selected as

$$T_1 = 0.0689908 \,, \qquad (23e)$$

the term preceding a_f becomes

$$b_{11}/240 + (T_1 s + 1)b_{12}$$
$$= s^3 + 49.132s^2 + 1250s + 12500 \cong P(s) \,. \qquad (23f)$$

Also define a_f such that

$$A_f(s)a_f = B_f(s)a_r^* \,,$$
$$B_f(0)/A_f(0) = 1 - e_1 \,. \qquad (23g)$$

Then Eq. (23d) becomes

$$a = [B_f(s)/A_f(s) + e_1 P(0)/P(s)]a_r^* \,. \qquad (24a)$$

The steady state value of a defined as a_{ss} is

$$a_{ss} = a_r^* \,. \qquad (24b)$$

The steady state value of u_a defined as u_{ass} is obtained from Eqs. (23c, g) as

$$u_{ass} = (1/240)(1 - e_1)a_r^* \,. \qquad (24c)$$

The steady state value of α defined as α_{ss} is obtained from Eq. (15) as

$$\alpha_{ss} = (1/671.4)e_1 a_r^* \,. \qquad (24d)$$

Thus a controller of the following characteristics is realized.

(1) The normal acceleration a follows its command a_r^*.

(2) The transient can be freely adjusted by $B_f(s)/A_f(s)$.

(3) The steady state value u_{ass} and α_{ss} can be freely adjusted by e_1.

The combined feedback and feed-forward controller will be expressed as

$$A_c^*(s)[u_a; u_r] = B_a^*(s)a_r^* - B_c(s)[a; q; \alpha] \,,$$
$$A_c^*(s) = [A_f(s)\ \ 0; -240k_0(T_1 s + 1)\ \ s] \,,$$
$$B_c(s) = [0\ \ 0\ \ 0; k_1 s + k_0\ \ k_2 s\ \ 0] \,,$$
$$B_a^*(s) = [B_f(s)/240; k_0 e_1] \,. \qquad (25)$$

Simulation is performed with the designed controller Eq. (25), where the exact model Eq. (13) is used. Three examples are considered, and the results match to the expectation. The first example is "No feed-forward control", where

$$A_f(s) = 1,\ B_f(s) = 0,\ e_1 = 1 \,. \qquad (26a)$$

The second example is "Quick response feed-forward", where

$$A_f(s) = s^2 + 32s + 365,\ B_f(s) = 16s,\ e_1 = 1 \,. \qquad (26b)$$

The third example is "Reduced angle-of-attack feed-forward", where

$$A_f(s) = s + 25,\ B_f(s) = 12.5,\ e_1 = 0.5 \,. \qquad (26c)$$

7. MIMO DESIGN

Various aspects of MIMO design will be discussed. The topics are as follows.

(1) The mathematical difficulties in MIMO design
(2) Excessive freedom
(3) CAD for polynomial
(4) Design procedure

Simply stated, the mathematical definition of MIMO design may be as follows.

"For given $A_p(s)$, $B_p(s)$, and $P(s)$, find $A_c(s)$ and $B_c(s)$ such that

$$A_c(s)A_p(s) + B_c(s)B_p(s) = A(s) \,, \qquad (27a)$$
$$\det A(s) = P(s) \,. \qquad (27b)$$

The first difficulty is that $A(s)$ can not be determined from det $A(s)$. There are too much freedom left to designer in assigning $A(s)$ from det $A(s)$. The second difficulty is that even though $A(s)$ is determined, there is difficulty in solving DEQ (27a). DEQ can be expressed by the linear relation between controller parameters and the parameters of $A(s)$, but the number of equations is usually less than that of controller parameters, and matrix dimension is large.

These difficulties may be called difficulty of excessive freedom. Excessive freedom is a basic nature of MIMO quite different from SISO. To use this freedom wisely, designers are required to have good design common sense based on practical design experience.

In CDM application to MIMO, mathematical manipulation of polynomial matrix like addition, subtraction, multiplication, and inversion is required. Also solution of Diophantine equation must be expedited . For these purpose, a good CAD for polynomial is essential and the recent development in this direction is very encouraging (Kwakernaak, 2000).

The MIMO design steps taken in this paper are as follows.
(1) The plant model given in state-space is simplified and expressed in polynomial matrices of right co-prime fraction form.
(2) The transfer functions between various inputs and outputs are calculated, and the denominator and numerator polynomials are shown on coefficient diagrams. Appropriate input and output are selected for feedback control design, where the system becomes SIMO.
(3) After the design, the feedback controller is incorporated with the plant, and a new plant is defined.
(4) Repeat Step (2)(3) until the desired characteristic polynomial is obtained.
(5) Feed-forward control is designed with the unused excess freedom.

By this procedure, $A_c(s)$ and $B_c(s)$ are designed step by step and excess freedom is taken care of at each stage. The closed-loop system polynomial matrix $A(s)$ are finally obtained together with $A_c(s)$ and $B_c(s)$. Simultaneous design of closed-loop system and controller is an important feature of CDM.

9. CONCLUSION

The major results of this paper are as follows.
(1) At control structure design, selection of input and output is very important. For this purpose, the coefficient diagrams of denominator and numerator polynomials are found to be very effective.
(2) MIMO problem (matrix DEQ) can be decomposed into a series of SIMO problems (scalar DEQ). The feedback control with desired characteristic polynomial can be effectively designed with this approach.
(3) The excess freedom can be utilized for feed-forward control design, whereby various control characteristics can be easily obtained.
(4) Effective CAD for polynomial matrix is essential.

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