15th IFAC Symposium on Automatic Control in Aerospace
September 2-7, 2001, Bologna/Forli, Italy

APPLICATION OF COEFFICIENT DIAGRAM METHOD TO DUAL-CONTROL-SURFACE MISSILE

Shunji Manabe

1-8-12 Kataseyama, Fujisawa, Kanagawa 251-0033, Japan

Abstract: The normal acceleration control of dual-control-surface missile is a typical MIMO problem, where various modern control design techniques are employed. Coefficient Diagram Method (CDM), with proven effectiveness in SISO or SIMO control design, has been applied to this MIMO problem. It is shown that MIMO design problem can be decomposed into series of SIMO problems, and feedback controller can be designed step by step with CDM. The extra freedom in design is used to design feed-forward controller of various characteristics. The meaning of extra freedom typical in MIMO system is clarified.

Keywords: Control system design, Control theory, Missiles, MIMO, Polynomials.

1. INTRODUCTION

The normal acceleration control of the dual-control-surface missile is a typical MIMO (Multi-input-multi-output) problem, where various modern control design techniques are employed. The Coefficient Diagram Method (CDM) is proven to be effective in SISO (Single-input-single-output) or SIMO problem (Manabe, 1998b). This paper shows CDM can be applied to MIMO by decomposing the problem into series of SIMO problems. Then each SIMO problem can be solved by standard CDM procedure.

Contrary to SISO case, too much design freedom is left to designer in MIMO case, and to make wise decision at the design becomes a difficult problem to the designer. Serial solution of SIMO problem greatly helps the designer to make such decision wisely. By this way, the meaning of normal acceleration control of the dual-control-surface missile becomes much clearer compared with the controller design by modern control.

This paper is organized as follows. In Section 2, the basics of CDM are explained. In Section 3, the mathematical model of a dual-control-surface missile is presented. In Section 4, the basic control structure is determined based on coefficient diagram analysis. In Section 5, a feedback controller is designed as a SIMO problem by CDM. In Section 6, various feed-forward controllers are suggested. In Section 7, simulation results are shown for these controllers. In Section 8, the CDM design of MIMO is summarized.

2. BASICS OF CDM

2.1 Basic Philosophy of CDM

The CDM is an algebraic control design approach with the following five features (Manabe, 1998b).

(1) Polynomials and polynomial matrices are used for system representation.
(2) Characteristic polynomial and controller are simultaneously designed.
(3) Coefficient diagram is effectively utilized.
(4) The sufficient condition for stability by Lipatov (1978) constitutes the theoretical basis of CDM (Manabe, 1999).
(5) Kessler (1960) standard form is improved and used as the standard form of CDM.

CDM design is based on the stability index and equivalent time constant as defined later. Thus for the specified settling time, a controller of the lowest order with the narrowest bandwidth and of no-overshoot can be easily designed. CDM can be considered as "Generalized PID", because the controller can be more complex than PID, and more reliable parameter selection rules are provided. Also CDM can be considered as "Improved LQG", because the order of controller is smaller and weight selection rules are also given (Manabe, 1998a).

2.2 Mathematical Model

The standard block diagram of the CDM design is shown in Fig. 1. This diagram is valid to SISO, SIMO, and MIMO. In SISO, the variables and components are all scalars but in MIMO they are vectors and matrices of proper dimension. The plant equation is given as

$$A_p(s)x = u + d \tag{1a}$$

$$y = B_p(s)x\,, \tag{1b}$$

where u, y, and d are input, output, and disturbance. The symbol x is called the basic state variable. $A_p(s)$ and $B_p(s)$ are the denominator and numerator polynomial matrix of the plant. It will be easily seen that this expression has a direct correspondence with the control canonical form of the state-space expression, and x corresponds to the state variable of the lowest order. All the other states are expressed as the derivatives of x of high order.

Controller equation is given as

$$A_c(s)u = B_a(s)y_r - B_c(s)(y+n)\,, \tag{2}$$

where y_r and n are reference input and noise on the output. $A_c(s)$ is the denominator polynomial matrix of the controller. $B_a(s)$ and $B_c(s)$ are called the reference and feedback numerator polynomial matrix of the controller. Because the controller transfer function has two numerators, it is called two-degree-of-freedom system. This expression corresponds to the observer canonical form of the state-space expression.

Elimination of y and u from Eq. (2) by Eqs. (1a, b) gives

$$A(s)x = B_a(s)y_r + A_c(s)d - B_c(s)n\,, \tag{3a}$$

where $A(s)$ is the closed-loop system polynomial matrix and given as

$$A(s) = A_c(s)A_p(s) + B_c(s)B_p(s)\,. \tag{3b}$$

The characteristic polynomial $P(s)$ is given as,

$$P(s) = \det A(s)\,. \tag{3c}$$

The input output relation is given as

$$\begin{bmatrix} x \\ y \\ u \end{bmatrix} = \frac{1}{P(s)} \begin{bmatrix} I \\ B_p(s) \\ A_p(s) \end{bmatrix} \operatorname{adj} A(s)$$

$$[B_a(s)y_r + A_c(s)d - B_c(s)n] - \begin{bmatrix} 0 \\ 0 \\ d \end{bmatrix}. \tag{4}$$

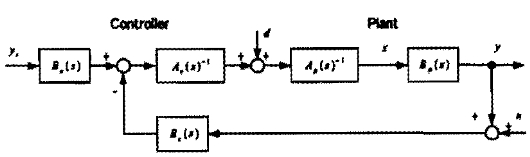

Fig. 1. CDM standard block diagram

2.3 Basic Relations

Some mathematical relations extensively used in CDM will be introduced hereafter. These relations will be freely used in later sections. The characteristic polynomial $P(s)$ is given in the following form.

$$P(s) = a_n s^n + \cdots + a_1 s + a_0 = \sum_{i=0}^{n} a_i s^i \tag{5}$$

The stability index γ_i, the equivalent time constant τ, and the stability limit γ_i^* are defined as follows.

$$\gamma_i = a_i^2/(a_{i+1}a_{i-1})\,, \qquad i = 1 \sim n-1\,, \tag{6a}$$

$$\tau = a_1/a_0\,, \tag{6b}$$

$$\gamma_i^* = 1/\gamma_{i+1} + 1/\gamma_{i-1}\,, \tag{6c}$$

γ_n and γ_0 are defined as ∞.

The equivalent time constant of the i-th order τ_i is defined in the similar manner as τ.

$$\tau_i = a_{i+1}/a_i \tag{7}$$

By Eqs. (6a) and (7),

$$\tau_i/\tau_{i-1} = (a_{i+1}/a_i)(a_i/a_{i-1}) = 1/\gamma_i\,, \tag{8a}$$

$$\tau_i = \tau_{i-1}/\gamma_i = \tau/(\gamma_i \cdots \gamma_2\gamma_1)\,. \tag{8b}$$

By repeated use of Eqs. (7) a_i is expressed by τ_i and a_0.

$$a_i = \tau_{i-1} \cdots \tau_1 \tau a_0 \tag{9a}$$

By Eq. (8b), this reduces to

$$a_i = a_0 \tau^i/(\gamma_{i-1}\gamma_{i-2}^2 \cdots \gamma_2^{i-2}\gamma_1^{i-1})\,, \quad i \ge 2\,. \tag{9b}$$

Then characteristic polynomial is expressed by a_0, τ, and γ_i as follows.

$$P(s) = a_0 [\{\sum_{i=2}^{n} (\prod_{j=1}^{i-1} 1/\gamma_{i-j}^{j})(\tau s)^i\} + \tau s + 1] \tag{9c}$$

The sufficient condition for stability and instability constitutes the theoretical basis of CDM. It states as follows.

"The system of any order is stable, if all the partial 4th order polynomials are stable with the margin of 1.12. The system is unstable if some partial 3rd order polynomial is unstable."

Thus the sufficient condition for stability is given as

$$a_i > 1.12\,[\frac{a_{i-1}}{a_{i+1}} a_{i+2} + \frac{a_{i+1}}{a_{i-1}} a_{i-2}]\,, \tag{10a}$$

$$\gamma_i > 1.12\gamma_i^*\,, \quad \textit{for all } i = 2 \sim n-2\,. \tag{10b}$$

The sufficient condition for instability is given as

$$a_{i+1}a_i \le a_{i+2}a_{i-1}\,, \tag{11a}$$

$$\gamma_{i+1}\gamma_i \le 1\,, \quad \textit{for some } i = 1 \sim n-2\,. \tag{11b}$$

These conditions are graphically expressed in the coefficient diagram, and the designer can intuitively assess the stability of the system (Manabe, 1998b).

In CDM, the following stability indices are recommended.

$$\gamma_{n-1} = \ldots = \gamma_3 = \gamma_2 = 2, \quad \gamma_1 = 2.5 \tag{12a}$$

For more relaxed form, with very small sacrifice of stability,

$$\gamma_i > 1.5\gamma_i^*, \quad i = n-1 \sim 4$$

$$\gamma_3 = \gamma_2 = 2, \quad \gamma_1 = 2.5\,. \tag{12b}$$

In these cases, the step response of Eq. (4) has no overshoot, and the settling time is about 2.5~3τ.

3. MATHEMATICAL MODEL OF DUAL-CONTROL-SURFACE MISSILE

The dual-control-surface missile is sown in Fig. 2. This example is selected from an open literature (Ochi, 1997). This missile has length of 5 m. The operating condition is 10,000 m in altitude and 3 M (900m/sec) in speed. The required normal acceleration is 25 g or about 250 m/sec^2. The state equation is given as

$$\begin{bmatrix} \dot{w} \\ \dot{q} \\ \dot{\delta}_f \\ \dot{\delta}_r \\ a \\ \alpha \end{bmatrix} = \begin{bmatrix} -0.746 & 900 & -240 & -240 & 0 & 0 \\ 0.0532 & -0.572 & 155 & -191 & 0 & 0 \\ 0 & 0 & -100 & 0 & 100 & 0 \\ 0 & 0 & 0 & -100 & 0 & 100 \\ 0.746 & 0.0991 & 240 & 240 & 0 & 0 \\ 1/900 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} w \\ q \\ \delta_f \\ \delta_r \\ u_f \\ u_r \end{bmatrix}, \tag{13}$$

where w is the downward speed in m/sec; q is the pitch rate in rad/sec; δ_f and δ_r are the deflection of the front and rear control surfaces in rad; a is the normal acceleration in m/sec^2; α is the angle of attack in rad and defined as w/U, where U is the forward speed. The actuator dynamics is represented by a time lag of 0.01 sec, and control inputs are u_f and u_r.

The simplified model for control design is deduced from this model. First, the actuator dynamics is neglected.

$$\delta_f = u_f, \quad \delta_r = u_r \tag{14a}$$

Then virtual input u_a is defined as

$$u_a = u_f + u_r . \tag{14b}$$

Then

$$u_f = u_a - u_r . \tag{14c}$$

By Eqs. (14a, b, c), the design model is derived as

$$\begin{bmatrix} \dot{w} \\ \dot{q} \\ a \\ q \\ \alpha \end{bmatrix} = \begin{bmatrix} -0.746 & 900 & -240 & 0 \\ 0.0532 & -0.572 & 155 & -346 \\ 0.746 & 0.0991 & 240 & 0 \\ 0 & 1 & 0 & 0 \\ 1/900 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} w \\ q \\ u_a \\ u_r \end{bmatrix}. \tag{15}$$

For easier manipulation, MATLAB style expression of vector is employed, where a column vector [1 3 6]^T is represented as [1; 3; 6]. Then the polynomial matrix expression is given as

$$A_m(s)[w;q] = B_u[u_a;u_r],$$
$$[a;q;\alpha] = B_m(s)[w;q],$$
$$A_m(s) = \begin{bmatrix} s+0.746 & -900 \\ -0.0532 & s+0.572 \end{bmatrix},$$
$$B_m(s) = \begin{bmatrix} -s & 900.0991 \\ 0 & 1 \\ 1/900 & 0 \end{bmatrix},$$
$$B_u = [-240 \;\; 0; \;\; 155 \;\; -346]. \tag{16}$$

CDM-standard polynomial matrix model is obtained as

$$A_p(s)[w_1;q_1] = [u_a;u_r],$$
$$[a;q;\alpha] = B_p(s)[w_1;q_1],$$
$$[w_1;q_1] = B_{u1}^{-1}[w;q], \quad B_{u1} = [-240 \;\; 0;0 \;\; -346],$$
$$w_1 = -0.0041667w = -3.75\alpha,$$
$$q_1 = -0.0028902q,$$
$$A_p(s) = B_u^{-1}A_m(s)B_{u1},$$
$$= \begin{bmatrix} s+0.746 & -1297.5 \\ 0.44798s+0.29729 & s-580.68 \end{bmatrix},$$
$$B_p(s) = B_m(s)B_{u1}.$$
$$= \begin{bmatrix} 240s & -311430 \\ 0 & -346 \\ -240/900 & 0 \end{bmatrix}. \tag{17}$$

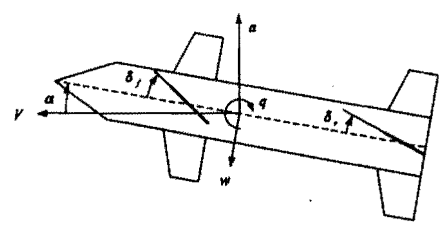

Fig. 2. Dual-control-surface missile

4. CONTROL STRUCTURE DESIGN

The purpose of control is to make the normal acceleration a to follow the command a_r. Thus the number of main output is only one, but there are 2 inputs and 3 outputs, and excessive freedom is left to designer. In order to determine the basic control structure, the 6 input-output relations are derived as

$$[a;q;\alpha] = [1/A_{pd}(s)]B_p(s)\,\mathrm{adj}\,A_p(s)[u_a;u_r],$$
$$A_{pd}(s) = \det A_p(s) = s^2 + 1.3180s - 47.453,$$
$$B_p(s)\,\mathrm{adj}\,A_p(s) = [baa \;\; bar; bra \;\; bqr; b\alpha a \;\; b\alpha r]$$
$$baa = 240s^2 + 152.64s + 92586,$$
$$bar = -34.289s - 232330,$$
$$bqa = 155s + 102.86, \quad bqr = -346s - 258.12,$$
$$b\alpha a = -0.266667s + 154.85, \quad b\alpha r = -346. \tag{18}$$

These result are obtained by Polynomial Toolbox (Kwakernaak, 2000); These polynomials are shown in the coefficient diagram of Fig. 3, where the coefficient of s^i is multiplied by 10^i for convenience. The minus value is indicated by (-).

For denominator $A_{pd}(s)$, the 1st order coefficient is too small and the 0th order coefficient is negative. These two coefficients must be modified by the feedback. The 1st order coefficient can be modified by proper feedback involving bqa and bqr. Because these two signals are almost alike, either of them can be used. The 0-th order coefficient can be modified by bar, $b\alpha a$, and $b\alpha r$. However baa can not be used, because it affects both 2nd and 0-th order.

From this analysis, the most reasonable feedback control is to control u_r by a and q, while keeping u_a equal to zero.

The signal a and α are similar in nature. Since a is necessary for control anyway, there is no need for α in feedback control. As explained later, α can be controlled effectively by feed-forward control.

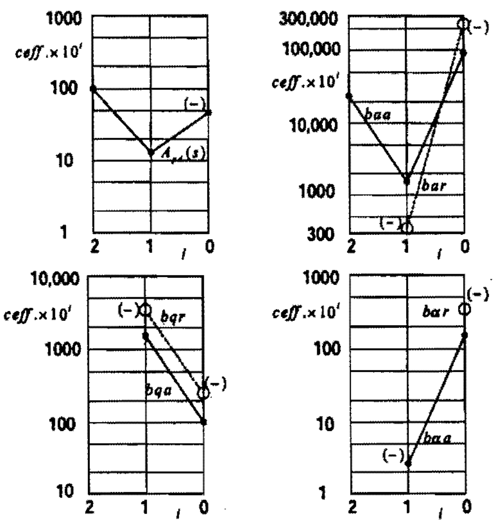

Fig. 3. Denominator and numerator polynomial

5. FEEDBACK CONTROL DESIGN

By making $u_a = 0$, the plant becomes a SIMO system and expressed as

$$[a;q;\alpha]=[1/A_{pd}(s)][bar;bqr;b\alpha r]\,u_r\,. \qquad (19a)$$

The natural choice of controller is a PID controller with the feedback of q and a such as

$$su_r = k_0 a_r - [k_2 sq + (k_1 s + k_0)a]\,. \qquad (19b)$$

The block diagram is shown in Fig. 4. From Eqs. (3b, c), the characteristic polynomial $P(s)$ becomes

$$P(s) = sA_{pd}(s) + k_2 s\,(bqr) + (k_1 s + k_0)\,(bar)$$

$$= s^3 + a_2 s^2 + a_1 s + a_0\,. \qquad (19c)$$

This is a 3rd order system, and γ_2 and γ_I are chosen as the standard value.

$$\gamma_2 = 2,\quad \gamma_1 = 2.5 \qquad (20a)$$

Because the neglected actuator dynamics has time constant of 0.01 sec, the good value of τ_2 is 0.02 sec, twice as large. By Eq. (8b),

$$\tau = 0.1\,. \qquad (20b)$$

Then by Eq. (9b),

$$a_2 = 50,\quad a_1 = 1250,\quad a_0 = 12500 \qquad (20c)$$

By solving Diophantine equation, Eq. (19c), controller parameters are obtained as

$$k_2 = -0.140162,\quad k_1 = -0.0054209,$$

$$k_0 = -0.053803\,. \qquad (20d)$$

The solution is easily obtained by CDM CAD (MSS, 2000). The coefficient diagram is shown in Fig. 5.

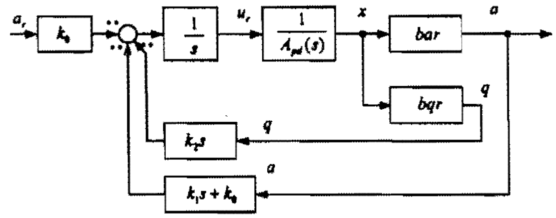

Fig. 4. SIMO block diagram

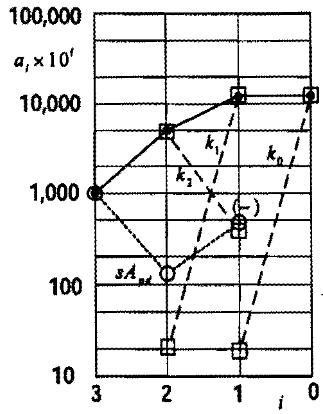

Fig. 5. Coefficient diagram of SIMO design

6. FEED-FORWARD CONTROL DESIGN

The closed-loop polynomial matrix equation, such as Eqs. (1b) and (3a), can be obtained from Eqs. (17) and (19b) as

$$A(s)[\alpha;q] = B_a(s)[u_a;a_r]\,,$$

$$[a;q;\alpha] = B_p(s)[\alpha;q]\,,$$

$$A(s) = A_c(s)A_p(s) + B_c(s)B_p(s)\,,$$

$$A_c(s) = [1\ 0;0\ s]\,,$$

$$B_c(s) = [0\quad 0\quad 0;\quad k_1 s + k_0\quad k_2 s\quad 0]\,,$$

$$B_a(s) = [1\ 0;0\ k_0]\,. \qquad (21)$$

Then as Eqs. (17) and (18), six input-output relations are obtained as

$$[a;q;\alpha] = [1/P(s)]\,\mathrm{adj}A(s)\ B_a(s)[u_a;a_r]\,,$$

$$P(s) = \det A(s) = s^3 + 50s^2 + 1250s + 12500\,,$$

$$\mathrm{adj}A(s)\ B_a(s) = [b_{11}\ b_{12};b_{21}\ b_{22};b_{31}\ b_{32}]\,,$$

$$b_{11} = 240s^3 + 11792s^2 + 92586s\,,$$

$$b_{12} = 1.8448s + 12500\,,$$

$$b_{21} = -295.15s^2 - 4364.9s\,,$$

$$b_{22} = 18.616s + 13.887\,,$$

$$b_{13} = -0.266667s^2 - 308.29s - 4468.3\,,$$

$$b_{32} = 18.616\,. \qquad (22)$$

When only a is considered as output, relation becomes

$$a = [1/P(s)](b_{11}\,u_a + b_{12}\,a_r)\,. \qquad (23a)$$

The polynomial b_{11} is expressed as

$$b_{11} = 240(s^3 + 49.132s^2 + 385.77s)\,. \qquad (23b)$$

The 3rd and 2nd order coefficients are very similar to those of $P(s)$. The polynomial b_{12} is almost equal to $P(0)$. Now a feed-forward controller is assumed as

$$[u_a;a_r]=[1\ \ 0;240(T_1s+1)\ \ 1][a_f/240;e_1a_r^*]. \tag{23c}$$

Eq. (23a) becomes

$$a=[1/P(s)][\{b_{11}/240+(T_1s+1)b_{12}\}a_f+b_{12}\,e_1a_r^*]. \tag{23d}$$

When T_1 is selected as

$$T_1=0.0689908\,, \tag{23e}$$

the term preceding a_f becomes

$$b_{11}/240+(T_1s+1)b_{12}$$
$$=s^3+49.132s^2+1250s+12500\cong P(s). \tag{23f}$$

Also define a_f such that

$$A_f(s)a_f=B_f(s)a_r^*,$$
$$B_f(0)/A_f(0)=1-e_1. \tag{23g}$$

Then Eq. (23d) becomes

$$a=[B_f(s)/A_f(s)+e_1P(0)/P(s)]a_r^*. \tag{24a}$$

The steady state value of a defined as a_{ss} is

$$a_{ss}=a_r^*. \tag{24b}$$

The steady state value of u_a defined as u_{ass} is obtained from Eqs. (23c, g) as

$$u_{ass}=(1/240)(1-e_1)a_r^*. \tag{24c}$$

The steady state value of α defined as α_{ss} is obtained from Eq. (15) as

$$\alpha_{ss}=(1/671.4)e_1a_r^*. \tag{24d}$$

Thus a controller of the following characteristics is realized.

(1) The normal acceleration a follows its command a_r^*.

(2) The transient can be freely adjusted by $B_f(s)/A_f(s)$.

(3) The steady state value u_{ass} and α_{ss} can be freely adjusted by e_1.

The combined feedback and feed-forward controller will be expressed as

$$A_c^*(s)[u_a;u_r]=B_a^*(s)a_r^*-B_c(s)[a;q;\alpha],$$
$$A_c^*(s)=[A_f(s)\ \ 0;-240k_0(T_1s+1)\ \ s],$$
$$B_c(s)=[0\ \ 0\ \ 0;k_1s+k_0\ \ k_2s\ \ 0],$$
$$B_a^*(s)=[B_f(s)/240;k_0e_1]. \tag{25}$$

7. Simulation

Simulation is performed with the designed controller Eq. (25), where the exact missile model Eq. (13) is used. Three examples are shown.

<u>Example 1. No feed-forward control</u>

$$B_f(s)=0,\quad A_f(s)=1,\quad e_1=1,\quad a_r^*=300\,m/s^2.$$

The result is shown in Fig.6. The left-up figure is for normal acceleration a, and left-down for pitch rate q. The right-up figure is for the reference angle of attack α_r (solid line), obtained by Eq. (24d), and angle of attack α (broken line). The right-down figure is for control surface control commands; u_a (solid line), u_f (broken line), and u_r (dotted line). The a and α settle to the command value in 0.3 sec or 3τ. The response is very smooth. Because u_a is zero, u_f and u_r swing to the opposite direction. The maximum deflection is 0.28 rad or 16 deg.

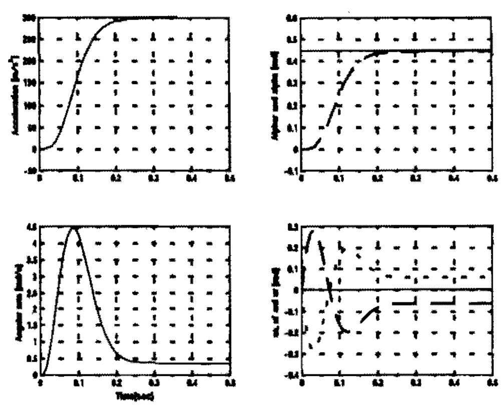

Fig. 6. No-feed-forward control

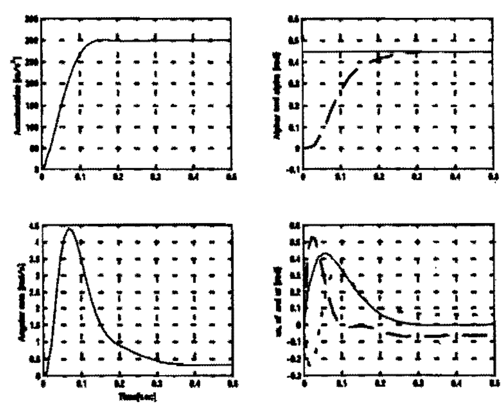

Fig. 7. Quick response feed-forward

<u>Example 2. Quick response feed-forward</u>

$B_f(s)=16s,\ A_f(s)=s^2+32s+365,\ e_1=1,$

$a_r^*=300\,m/s^2$. The result is shown in Fig.7. The a settles to the command value in 0.15 sec, but the α takes 0.3 sec to settle. This quick response is realized by a large deflection of u_f, 0.52 rad or 30 deg.

<u>Example 3. Reduced angle-of-attack feed-forward</u>

$B_f(s)=12.5,\ A_f(s)=s+25,\ e_1=0.5,$

$a_r^*=300\,m/s^2$. The result is shown in Fig.8. The u_a carries half the acceleration. The deflection of angle of attack α now becomes half form 0.44 rad to 0.22 rad or 13 deg. The maximum deflection of u_f is 0.44 rad or 25 deg. Both u_f and u_r have steady state value of 0.3 rad or 17 deg.

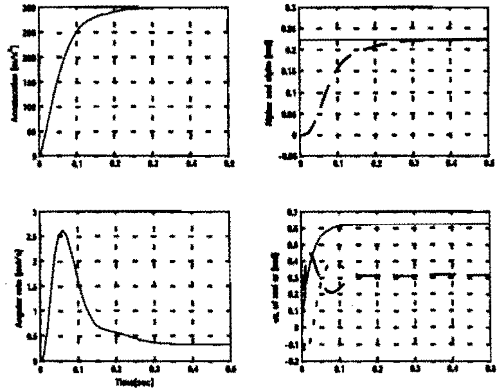

Fig. 8. Reduced angle-of attack feed-forward

These examples show the variety of control features obtained by the choice of feed-forward control for the same characteristic polynomial. This is the typical MIMO feature and quite different from SISO case.

8. SUMARY OF CDM-MIMO DESIGN

The MIMO design steps taken in this paper are as follows.

(1) The plant model given in state-space is simplified and expressed in polynomial matrices of right co-prime fraction form.

(2) The transfer functions between various inputs and outputs are calculated, and the denominator and numerator polynomials are shown on coefficient diagrams. Appropriate input and output are selected for feedback control design, where the system becomes SIMO.

(3) After the design, the feedback controller is incorporated with the plant, and a new plant is defined.

(4) Repeat Step (2)(3) until the desired characteristic polynomial is obtained.

(5) Feed-forward control is designed with the unused excess freedom.

9. CONCLUSION

The major results of this paper are as follows.

(1) At control structure design, selection of input and output is very important. For this purpose, the coefficient diagrams of denominator and numerator polynomials are found to be very effective.

(2) MIMO problem can be decomposed into a series of SIMO problems. The feedback control with desired characteristic polynomial can be effectively designed with this approach.

(3) The excess freedom can be utilized for feed-forward control design, whereby various control characteristics can be easily obtained.

REFERENCES

Hori, Y. (1994). 2-mass system control based on resonance ratio control and Manabe polynomials. Proc. of First Asian Control Conference, Tokyo, July 27-30, 1994, Vol. 3, III-741-744.

Kessler, C. (1960). Ein Beitrag zur Theorie mehrschleifiger Regelungen. *Regelungstechnik*, **8**, **8**, 261-266.

Kim, Y. C. and others (2000). Application of CDM to MIMO systems: control of hot rolling mill. 3rd Asian Control Conference, July 4-7, 2000, Shanghail, 2091-2096.

Kitamori, T. (1979). A method of control system design based upon partial knowledge about controlled processes. *Transaction of SICE*, **15**, 4, 549-555.

Kwakernaak, H. and M. Sebek (2000). PolyX home page. http://www.polyx.cz/

Lipatov, A. V. and N. I. Sokolov (1978). Some sufficient conditions for stability and instability of continuous linear stationary systems. translated from *Automatika i Telemekhanika*, No. 9, pp. 30-37, 1978; in *Automat. Remote Contr.*, 39, 1285-1291, 1979.

Manabe, S. (1994). Coefficient diagram method as applied to the attitude control of controlled-bias-momentum satellite. 13th IFAC Symposium on Automatic Control in Aerospace, Sept. 12-16, 1994, Palo Alto, CA, 322-327.

Manabe, S. (1997). The application of coefficient diagram method to the ACC benchmark problem. 2nd Asian Control Conference, July 22-25, 1997, Seoul, II-135-138.

Manabe, S. (1998a). Analytical weight selection for LQ design. 8th Workshop on Astrodynamics and Flight Mechanics, Sagamihara, ISAS, July 23-24, 1998.

Manabe, S. (1998b). The coefficient diagram method. 14th IFAC Symposium on Automatic Control in Aerospace, Aug. 24-28, 1998, Seoul, Korea, pp. 199-210.

Manabe, S. (1999). Sufficient condition for stability and instability by Lipatov and its application to the coefficient diagram method. 9-th Workshop on Astrodynamics and Flight Mechanics, Sagamihara, ISAS, July 22-23,1999.

Manabe, S and Y. C. Kim (2000). Recent development of coeffcient diagram method. 3rd Asian Control Conference, July 4-7, 2000, Shanghail, 2055-2060.

MSS (2000). Mitsubishi Space Software Home Page. http://www.mss.co.jp/techinfo/cdm-cad/cdm_progrm.htm

Ochi, Y. and K. Kanai (1997). Normal acceleration control for an innovative missile via angle-of-attack control. AIAA-97-3686, Proc. of AIAA GNC Conf., 980-988.