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A Low-Cost Inverted Pendulum System for Control System Education

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Abstract. A low-cost inverted pendulum system for educational purpose is introduced. This system is low-cost (21,000Yens) and simple (only angle sensor is used). The controller is a second order unstable controller with one unstable pole. A design approach called as coefficient diagram method is used in design. The effectiveness of this approach is fully demonstrated by easiness of design, simplicity of the designed controller, and the robustness of the system.

Key Words Inverted pendulum; control system design; control theory; education; unstable controller; controllers

1. INTRODUCTION

The first purpose of this paper is to introduce a low-cost inverted pendulum system which has been used in many phases of control system education. The second purpose is to show the effectiveness of the coefficient diagram method (CDM) in control system design. The design of the inverted pendulum system serves as an illustrative example.

The inverted pendulum is a very convenient tool in control engineering education because of the following reasons;

(1) It attracts the attention of freshman students and motivates them for further study.
(2) It gives the students a good opportunity to apply control theory to the actual system.
(3) The experience of building an inverted pendulum gives the students a balanced knowledge of the total control system. They learn components such as sensors, actuators and controllers, and also learn the procedures in design, manufacturing, and testing.

An ordinary inverted pendulum consists of a cart on a rail, an inverted pendulum connected to an incremental encoder, a personal computer with appropriate interface cards, software, a power amplifier, and a driving motor with incremental encoder whose shaft is connected to the cart through appropriate mechanism. Thus the total system is fairly complicated and its total cost ranges around several hundred thousand Yens. It is not well suited to the day-to-day educational environment.

The proposed inverted pendulum consists of a toy model car with a toy motor and a 20 cm inverted pendulum connected to a contactless magnetic-type potentiometer, and an analog controller with three ICs containing 12 operational amplifiers and 4 transistors, which is driven by dry batteries. The system is light weight and portable, and easily demonstrated in a classroom. The system is very inexpensive and its material cost is only around 21,000 Yens.

This paper will first explain CDM, and the controller will be designed by CDM. Then the description of the total system will be made and its operation will be briefly explained.

2. COEFFICIENT DIAGRAM METHOD

The coefficient diagram is a semi-log diagram where the coefficients of characteristic polynomials are shown in logarithmic scale in the ordinate and the numbers of power corresponding to each coefficient are shown in the abscissa. The degree of convexity is a measure of stability. The general inclination of the curve is a measure of response speed. The variation of the shape of the curve is a measure of robustness. Thus the three major characteristics of control system, namely stability, response, and robustness are shown graphically in a single diagram, enabling the designer to make a balanced judgment in the course of his design.

The power of the coefficient diagram method (CDM) lies in that it generates not only non-minimum phase controllers but also unstable controllers when required. LQG fails to produce a robust controller for plant with flexibility (poles at the vicinity of the imaginary axis) as pointed by various authors (Edmunds, 1983, and Mills, 1992). CDM produces very robust controllers in such cases. The experiences show that only well-designed H^{∞} controller can be equivalent to CDM controllers.

There are three roots from which CDM has evolved. Late 1950's, Kessler (1960) made intensive efforts to establish synthesis (design) procedures for multi-loop control systems, and came out with a standard form, commonly called "Kessler Canonical Multi-loop Structure". The proposed system has been widely accepted in the steel mill industry. The CDM is simply the sophistication and generalization of Kessler's work.

Stability of control systems can be analyzed by Routh or Hurwitz criterion utilizing coefficients of characteristic polynomials. However in this way the effect of the variation of coefficients on stability is not clearly seen. Lipatov (1978) proposed sufficient conditions for stability and instability. Because of its simplicity, the relation of stability and instability with respect to the coefficients of the characteristic polynomials becomes very clear. These conditions are integrated to the design procedures of CDM.

In control system design, classical control theory and modern control theory are widely used. But there is other approach called algebraic approach, and Chen (1987) proposed a simple design approach based on this philosophy. His approach is basically sophistication of the pole allocation method for closed loop characteristic polynomials. Some of his idea constitutes basic philosophy of CDM. Although rational functions are commonly used in algebraic approach, only polynomials are used in CDM. In this way, design procedures are much simplified and become more straight forward.

Simply stated, CDM is an algebraic approach using only polynomials, where the coefficient diagram is utilized as a vehicle to collectively express the important features of the system, and an improved version of Kessler's standard form and the stability condition of Lipatov constitute the theoretical basis.

3. CONTROLLER DESIGN

Although the system is simple and consists of low-cost parts, it has such high performance characteristics as described below;

(1) The inverted pendulum keeps standing even when error exists in angle sensor or the cart is placed on a slope.
(2) The velocity of the cart follows the velocity reference signal, while the pendulum is kept in the upright position.
(3) The pendulum is raised to the upright position from the rest position automatically at the initiation of control.
(4) The above performance is realized by utilizing only one angle sensor. No position sensor is used.
(5) The cart operates on not-smooth surface with large disturbance force keeping the pendulum in position. This characteristics has been realized by a simple velocity feedback where the velocity signal is estimated by subtracting the internal voltage drop from the terminal voltage of the driving motor.

The inverted pendulum system is shown in Fig. 1. If the cart velocity is controlled by a velocity controller, the equation of motion of the pendulum becomes

$$\ddot{\phi} - a\,\phi = b\,\hat{v} \qquad (1)$$

$$a = b\,g = 73.5, \quad b = 3/(4\,L) = 7.5 \qquad (2)$$

where v is the velocity of the cart in m / sec and ϕ is the angle of the pendulum in radian. The half length of the pendulum L is 0.1 meter.

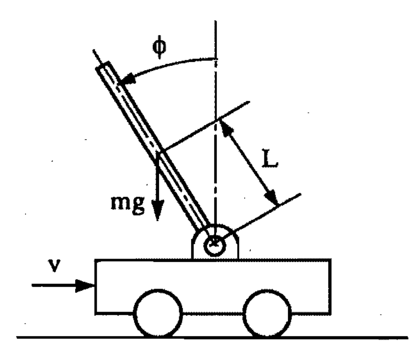

Fig. 1 Inverted pendulum system

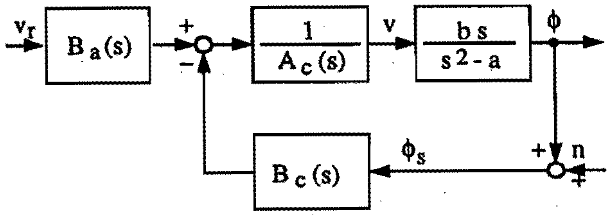

Fig. 2 Control system

Then the control system is shown in Fig. 2, where the controller is expressed in two-degree-of-freedom and polynomial style. The v_r is the velocity reference and n is the error of the angle sensor. The response is shown as follows;

$$\begin{bmatrix} \phi \\ v \end{bmatrix} = \frac{1}{P(s)} \begin{bmatrix} b\,s \\ (s^2 - a) \end{bmatrix} [B_a(s)\,v_r - B_c(s)\,n] \qquad (3)$$

where the closed loop characteristic polynomial P(s) is expressed as

$$P(s) = (s^2 - a)\,A_c(s) + b\,s\,B_c(s) \qquad (4)$$

At the steady state $v = v_r$ and thc following relation must hold.

$$-a\,B_a(0) / P(0) = 1 \qquad (5)$$

Also v must be zero even with the sensor error n at the steady state. This necessitates

$$B_c(0) = 0 \qquad (6)$$

With these in consideration, a controller of the following form will become a natural choice.

$$A_c(s) = s^2 + l_1\,s + l_0 \qquad (7)$$
$$B_c(s) = k_2\,s^2 + k_1\,s \qquad (8)$$
$$B_a(s) = l_0 \qquad (9)$$

Then the characteristic polynomial P(s) will be

$$P(s) = a_4\,s^4 + a_3\,s^3 + a_2\,s^2 + a_1 s + a_0 \qquad (10)$$
$$a_4 = 1$$
$$a_3 = l_1 + b\,k_2$$
$$a_2 = l_0 - a + b\,k_1$$
$$a_1 = -a\,l_1$$
$$a_0 = -a\,l_0$$

Thus if the coefficients a_i's are wisely chosen, the controller parameters will be immediately obtained. Selection of a_i's is the main topic of CDM.

4. PARAMETER SELECTION BY CDM

In CDM there are three important parameters; namely stability index γ_i, equivalent time constant τ, and stability limit γ_i^*. They are defined as follows, where the order of the polynomial is n;

$$\gamma_i = a_i^2 / (a_{i+1}\,a_{i-1}) \qquad i = 1 \sim n-1 \qquad (11)$$
$$\tau = a_1 / a_0 \qquad (12)$$
$$\gamma_i^* = 1/\gamma_{i+1} + 1/\gamma_{i-1} \qquad (13)$$

γ_n and γ_0 are considered as infinity.

Kessler (1960) proposed all γ_i to be 2. Lipatov (1978) (Corollary 2) proved the sufficient condition for stability is

$$\gamma_i \geq 1.12\,\gamma_i^* \qquad (14)$$

In CDM the standard choice is

$$\gamma_1 = 2.5 \qquad (15)$$
$$\gamma_{n-1} = \dots = \gamma_3 = \gamma_2 = 2 \qquad (16)$$

Although it is strongly recommended to stick to $\gamma_3 = \gamma_2 = 2$, γ_i for $i \geq 4$ can be more arbitrarily chosen under the condition that

$$\gamma_i \geq 1.5\,\gamma_i^* \qquad (17)$$

Although the standard choice usually guarantees sufficient robustness, sometimes it may be necessary to increase some γ_i up to 4 for robustness.

The standard choice has very interesting characteristics.

(1) In the case that the order of the numerator of the closed loop transfer function is 0, the step response has no overshoot.
(2) The settling time is about $2.5\,\tau \sim 3\,\tau$. This settling time is the shortest for the same τ.
(3) The wave forms are almost the same irrespective to the order n of P(s), if they have the same τ.

In order to improve robustness, the following values are selected in design of this inverted pendulum.

$$\gamma_3 = \gamma_2 = 4, \qquad \gamma_1 = 2.5 \qquad (18)$$
$$a_2 = 2\,a = 147 \qquad (19)$$

This selection is the key in CDM, and it requires some trial and error approach using the coefficient diagram. Although this is the most important, it will not be further discussed for brevity. After this selection, a_i, l_i, and k_i are calculated as follows;

$$[a_4 \dots a_0] = [1 \;\; 24.25 \;\; 147 \;\; 222.8 \;\; 135.1] \quad (20)$$

$$l_1 = -3.031, \; l_0 = -1.838 \quad (21)$$

$$k_2 = 3.637, \; k_1 = 29.65 \quad (22)$$

For easiness of implementation, some mathematical manipulation is necessary, and the final controller becomes

$$v = \frac{1}{0.2818\,s - 1}\left[\frac{-v_r + 7.823\,\phi_s}{1.931\,s + 1} - (1.025\,s + 7.823)\,\phi_s\right]$$

$$\phi_s = \phi + n \quad (23)$$

The total system is shown Fig. 3. The controller is 2nd order and the second stage of the controller has an unstable pole.

5. SYSTEM AND OPERATION

The system consists of the cart and the controller. The cart is a toy model car and is driven by a toy dc motor. It carries a 20 cm inverted pendulum which is connected to a contactless magnetic-type potentiometer. Its material cost is 11,000 Yens of which 9,000 Yens go to the potentiometer.

The controller is constructed on A4 size wood board and is powered by 4 dry batteries, ± 9 v for operational amplifiers and ± 6 v for power transistors. On a circuit board, 3 IC's and 4 transistors are placed. Each IC has 4 operational amplifiers. The 1st IC is for angle signal amplification. The 2nd IC is for the velocity controller. The 3rd IC is for the main controller, which is a 2nd order controller with an unstable pole. Its material cost is 10,000 Yens.

The control panel has two switches for power, a switch for AUTO-RESET, a rotary switch for velocity reference, a rotary switch for loop-gain-

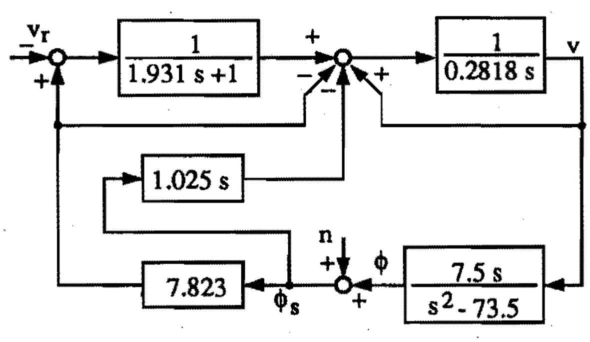

Fig. 3 Designed control system

variation of 2 - 1.5 - 1 - 0.7 - 0.5, and a variable resistor for zero adjust of angle signal which is only necessary for large deviation.

The system operation is as follows;

(1) The pendulum rise to upright position from the rest position at the activation of AUTO-RESET switch.
(2) The cart follows to the velocity reference.
(3) When the pendulum is being pushed, the cart moves, but if pushing is stopped the cart automatically returns to the original position.
(4) When one end of the table, on which the cart is placed, is raised, the cart moves to the other end, while the pendulum keeps the upright position. When the table is lowered the cart returns to the original position.
(5) When the loop gain is raised to 2 or lowered to 0.5, the system becomes unstable.
(6) The system is stable even the pendulum length is 7.5 cm or 40 cm (40 cm is mechanical limit).

6. CONCLUSION

This inverted pendulum has been successful in arousing interest of students, some of whom challenged to build the similar one. It is very moving to read their reports on their fascinating experience. The author strongly desires the similar effort be made by those interested for better control education.

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