

APPLICATION OF COEFFICIENT DIAGRAM METHOD TO MIMO DESIGN IN AEROSPACE

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Abstract: The longitudinal control of the fighter with dual control surfaces is a typical MIMO control problem, where various modern control design techniques are employed. Although Coefficient Diagram Method (CDM) is proven effective in SISO or SIMO control design, the concrete procedure for MIMO design is not established yet. A trial design by CDM is made for this MIMO problem and the result is compared with the standard H-inf or H2 design. *Copyright © 2002 IFAC*

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1. INTRODUCTION

At present, LQR and H-infinity are the most popular control design procedures for MIMO (Multi-Input-Multi-Output) system. However these methods are not up to expectation for practical application in aerospace community due to the following reasons.

- (1) Parameter tuning procedures are not provided.
- (2) Weight selection rules are not established.
- (3) The controller order is unnecessarily high.
- (4) Robustness is guaranteed only for predefined ones.
- (5) Some times, traditionally accepted good controllers are excluded.
- (6) Extension to gain scheduling or inclusion of proper saturation of state variable is difficult.

Thus classical control design by experienced engineer is still common in aerospace industry. However due to the difficulty in inheriting such experiences, some improvement is keenly needed.

The classical control and modern control are commonly used in control design, but there is a third approach generally called as algebraic design approach. The Coefficient Diagram Method (CDM) is one of the algebraic design approaches, where the coefficient diagram is used instead of Bode diagram, and the sufficient condition for stability by Lipatov constitutes its theoretical basis.

The CDM has been proved effective in number of examples, but they are SISO (Single-Input-Single-Output), or SIMO (Single-Input-Multi-Output) (Manabe, 1998b). The procedures for MIMO have not been established yet, and some trial designs are being made (Manabe, 2000). Usual approach is to decompose MIMO problems into series of SISO or SIMO problems, and to proceed the design by standard CDM procedure. In such decomposition, a good CAD system to handle polynomial matrix is indispensable, and Polynomial Toolbox developed in Europe has been a great help (Kwakernaak, 2000).

The purpose of this paper is to present one example of MIMO design by CDM. In order to make the comparison with other approaches, the problem is taken from the well-known example of the longitudinal control of a modern fighter in Robust Control Toolbox of MATLAB (Chiang, 1994).

This paper is organized as follows: In Section 2, the basics of CDM are briefly explained. In Section 3, the mathematical model and the problem statement are presented. In Section 4, the basic control structure is determined based on coefficient diagram analysis. In Section 5, feedback controllers are designed as SISO problems by CDM. In Section 6, simulation results are shown. In Section 7, the comparison is made with the H-inf controller.

2. BASICS OF CDM

The CDM is an algebraic control design approach with the following five features (Manabe, 1998b).

- (1) Polynomials and polynomial matrices are used for system representation.
- (2) Characteristic polynomial and controller are simultaneously designed.
- (3) Coefficient diagram is effectively utilized.
- (4) The sufficient condition for stability by Lipatov (1978) constitutes the theoretical basis of CDM (Manabe, 1999).
- (5) Kessler (1960) standard form is improved and used as the standard form of CDM.

CDM design is based on the stability index and equivalent time constant as defined later. Thus for the specified settling time, a controller of the lowest order with the narrowest bandwidth and of no-overshoot can be easily designed. CDM can be considered as "Generalized PID", because the controller can be more complex than PID, and more reliable parameter selection rules are provided. Also CDM can be considered as "Improved LQG", because the order of controller is smaller and weight selection rules are also given (Manabe, 1998a).

The characteristic polynomial $P(s)$ is given in the following form.

$$P(s) = a_n s^n + \dots + a_1 s + a_0 = \sum_{i=0}^n a_i s^i \quad (1)$$

The stability index γ_i , the equivalent time constant τ , and the stability limit γ_i^* are defined as follows.

$$\gamma_i = a_i^2 / (a_{i+1} a_{i-1}), \quad i = 1 \sim n-1, \quad (2)$$

$$\tau = a_1 / a_0, \quad (3)$$

$$\gamma_i^* = 1 / \gamma_{i+1} + 1 / \gamma_{i-1}, \quad (4)$$

γ_n and γ_0 are defined as ∞ .

The characteristic polynomial is expressed by a_0 , τ , and γ_i as follows:

$$P(s) = a_0 \left[\sum_{i=2}^n \left(\prod_{j=1}^{i-1} 1 / \gamma_{i-j}^j \right) (\tau s)^i \right] + \tau s + 1. \quad (5)$$

The sufficient condition for stability and instability constitutes the theoretical basis of CDM. It states as follows:

"The system of any order is stable, if all the partial 4th order polynomials are stable with the margin of 1.12. The system is unstable if some partial 3rd order polynomial is unstable."

Thus the sufficient condition for stability is given as

$$a_i > 1.12 \left[\frac{a_{i-1}}{a_{i+1}} a_{i+2} + \frac{a_{i+1}}{a_{i-1}} a_{i-2} \right], \quad (6a)$$

$$\gamma_i > 1.12 \gamma_i^*, \quad \text{for all } i = 2 \sim n-2. \quad (6b)$$

The sufficient condition for instability is given as

$$a_{i+1} a_i \leq a_{i+2} a_{i-1}, \quad (7a)$$

$$\gamma_{i+1} \gamma_i \leq 1, \quad \text{for some } i = 1 \sim n-2. \quad (7b)$$

These conditions are graphically expressed in the coefficient diagram, and the designer can intuitively assess the stability of the system (Manabe, 1998b).

In CDM, the following stability indices are recommended.

$$\gamma_{n-1} = \dots = \gamma_3 = \gamma_2 = 2, \quad \gamma_1 = 2.5. \quad (8a)$$

For more relaxed form, with very small sacrifice of stability,

$$\begin{aligned} \gamma_i &> 1.5 \gamma_i^*, \quad i = n-1 \sim 4 \\ \gamma_3 &= \gamma_2 = 2, \quad \gamma_1 = 2.5. \end{aligned} \quad (8b)$$

In these cases, the step response has no overshoot, and the settling time is about $2.5 \sim 3\tau$.

3. MATHEMATICAL MODEL AND PROBLEM STATEMENT

The problem selected is the longitudinal control of a modern fighter, shown in Fig. 1 (Chiang, 1994) (Safonov, 1981, 1988). This aircraft is trimmed at 25000 ft and 0.9 Mach. The linear model in state space expression is given as follows, where the MATLAB type expression is adopted, such that vector $[2 \ 4 \ 5]^T$ is expressed as $[2; 4; 5]$.

$$[\dot{\delta V}; \dot{\alpha}; \dot{q}; \dot{\theta}; \dot{\delta}_e; \dot{\delta}_c] = A_g [\delta V; \alpha; q; \theta; \delta_e; \delta_c] + B_g [u_e; u_c],$$

$$[\alpha; \theta] = C_g [\delta v; \alpha; q; \theta; \delta_e; \delta_c] + D_g [u_e; u_c],$$

$$A_g = \begin{bmatrix} -0.022567 & -36.617 & -18.897 & -32.090 & 3.2509 & -0.76257 \\ 9.2572e-5 & -1.8997 & 0.98312 & -7.2562e-4 & -0.17080 & -0.49652e-3 \\ 0.012338 & 11.720 & -2.6316 & 8.7582e-4 & -31.604 & 22.396 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -30 & 0 \\ 0 & 0 & 0 & 0 & 0 & -30 \end{bmatrix},$$

$$B_g = [0 \ 0; 0 \ 0; 0 \ 0; 0 \ 0; 30 \ 0; 0 \ 30],$$

$$C_g = [0 \ 1 \ 0 \ 0 \ 0 \ 0; 0 \ 0 \ 0 \ 1 \ 0 \ 0], \quad D_g = [0 \ 0; 0 \ 0]. \quad (9)$$

The state variables are velocity deviation (δV), angle of attack (α), attitude rate (q), attitude angle (θ), elevon angle (δ_e), and canard angle (δ_c). The output variables are α and θ . The control input variables are elevon actuator input (u_e) and canard actuator input (u_c).

By the use of two control inputs, the non-conventional precision flight path control becomes possible. Vertical translation mode keeps θ while varying α . Pitch pointing mode keeps both α and θ . Direct lift mode keeps α while varying θ . The stated objective of the control is interpreted as making α and θ to follow the respective commands (α_r and θ_r).

The more precise design specification is given in singular value specification as follows:

(1) Robustness Spec.: -40 dB/decade roll-off and at least -20 dB at 100 rad/sec.

(2) Performance Spec.: Minimize the sensitivity function as much as possible.

These specifications will be interpreted in the terms of CDM in the later section.

In order to make CDM MIMO design, the plant has to be expressed in a proper polynomial matrix fraction (PMF). In ordinary MIMO problem, right PMF is used for the plant and left PMF is used for the controller. But in this problem, left PMF is used for the plant, too. There are infinite numbers of left PMF, but only physically meaningful left PMF is to be used in CDM.

When actuator dynamics are moved to controller, the control inputs becomes δ_e and δ_c . Also q is replaced by $s\theta$, and δV is eliminated from the equation. The left PMF is given as follows.

$$A_p(s)[\alpha; \theta] = B_u(s)[\delta_e; \delta_c], \quad (10)$$

$$A_p(s) = \begin{bmatrix} a_{p11} & a_{p12} \\ a_{p21} & a_{p22} \end{bmatrix}, \quad B_u(s) = \begin{bmatrix} b_{u11} & b_{u12} \\ b_{u21} & b_{u22} \end{bmatrix},$$

$$a_{p11} = s + 1.9876,$$

$$a_{p12} = -0.0075030s^2 - 1.0029s + 0.00073219,$$

$$a_{p21} = -11.72s + 0.18730,$$

$$a_{p22} = s^3 + 2.6542s^2 + 0.29166s + 0.39591$$

$$b_{u11} = 0.066325, \quad b_{u12} = -0.17300,$$

$$b_{u21} = -31.604s - 0.67310,$$

$$b_{u22} = 22.396s + 0.49600.$$

In order to make design easier fictitious inputs δ_e^* and δ_c^* are introduced, such that

$$B_u(s)[\delta_e; \delta_c] = B_p(s)[\delta_e^*; \delta_c^*], \quad (11)$$

$$B_p(s) = \begin{bmatrix} 0 & b_{p12} \\ b_{p21} & 0 \end{bmatrix},$$

$$b_{p12} = b_{u12}, \quad b_{p21} = b_{u21}.$$

Then the following relation is derived.

$$[\delta_e; \delta_c] = E[\delta_e^*; \delta_c^*], \quad (12)$$

$$E = B_u^{-1}(s)B_p(s) = \begin{bmatrix} 1.3730 & 0.97298 \\ 0.52638 & 1.3730 \end{bmatrix} + \Delta(s).$$

In this equation $\Delta(s)$ can be neglected, because it is very small.

Thus the plant model for CDM design is obtained as follows:

$$A_p(s)[\alpha; \theta] = B_p(s)[\delta_e^*; \delta_c^*]. \quad (13)$$

Because the outputs of the designed controller are δ_e^* and δ_c^* , the following conversion is necessary to obtain actual control inputs u_e and u_c .

$$[u_e; u_c] = E[(s/30 + 1)\delta_e^*; (s/30 + 1)\delta_c^*]. \quad (14)$$

Because $B_p(s)$ is a simple skew diagonal form design becomes much simpler.

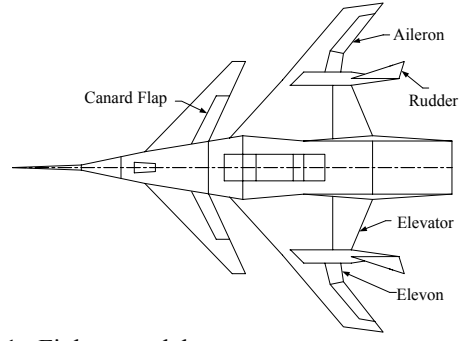


Fig. 1. Fighter model

4. CONTROL STRUCTURE DESIGN

The purpose of control is to make the outputs α and θ to follow the command α_r and θ_r . The specification given in terms of singular value can be interpreted as follows:

- (1) Each control channel should be independent and no interaction is expected.
- (2) Each channel should have the same characteristics.
- (3) The auxiliary sensitivity function of each channel should show -40 dB/decade roll-off and at least -20 dB at 100 rad/sec.

Usually the sensitivity function becomes larger when the interaction exists between two channels. Thus the minimization of sensitivity function makes the interaction the minimum. The singular value specification takes worse value between the two channels, and naturally each channel should show the same characteristics. In this situation, the two singular values take the same value and they are equal to the characteristics of each channel.

Thus the design process is divided into two phases, namely the basic design phase, where each control channel is designed, and the improvement phase, where the minimization of interaction is sought. For the basic design phase, coefficient diagram analysis is made as follows:

The characteristic polynomial for the original plant $P_0(s)$ are composed of diagonal component $P_{d0}(s)$ and skew component $P_s(s)$.

$$P_0(s) = \det(A_p(s)) = P_{d0}(s) + P_s(s), \quad (15a)$$

$$P_{d0}(s) = a_{p11}a_{p22}, \quad (15b)$$

$$P_s(s) = -a_{p12}a_{p21}. \quad (15c)$$

The coefficient diagram is shown in Fig. 2. Because of the large minus coefficient of s^2 of $P_s(s)$, the corresponding coefficient of $P_0(s)$ is minus and the open loop system is unstable. Now by proper feedback control, a_{p11} and a_{p22} are converted to a_{11} and a_{22} , such that

$$a_{11} = s + 15, \quad (16a)$$

$$a_{22} = s^3 + 15s^2 + 56.25s + 84.375. \quad (16b)$$

Then the new diagonal component $P_d(s)$ becomes

$$P_d(s) = a_{11}a_{22}, \quad (17a)$$

$$= s^4 + 30s^3 + 281.25s^2 + 928.13s + 1265.6,$$

and the characteristic polynomial $P(s)$ becomes

$$P(s) = P_d(s) + P_s(s), \quad (17b)$$

$$= s^4 + 29.912s^3 + 269.5s^2 + 928.32s + 1265.6.$$

As is clear from Fig. 2, $P(s)$ is almost equal to $P_d(s)$, and $P(s)$ displays a good convex shape, essential to good stability.

From the above observation, it is concluded that the controller can be designed for each channel independently, and the stability is automatically satisfied. For this design the standard CDM SISO design can be used. Actually Eqs. (16a, b) are derived from standard CDM approach.

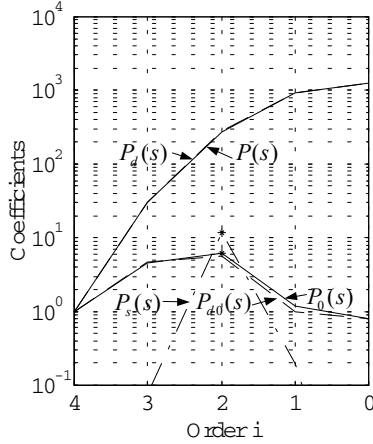


Fig. 2. Coefficient diagram

5. FEEDBACK CONTROL DESIGN

The plant equation is given in Eq. (13). It is repeated here again.

$$\begin{bmatrix} a_{p11} & a_{p12} \\ a_{p21} & a_{p22} \end{bmatrix} \begin{bmatrix} \alpha \\ \theta \end{bmatrix} = \begin{bmatrix} 0 & b_{p12} \\ b_{p21} & 0 \end{bmatrix} \begin{bmatrix} \delta_e^* \\ \delta_c^* \end{bmatrix}. \quad (18)$$

$$a_{p11} = s + 1.9876,$$

$$a_{p12} = -0.0075030s^2 - 1.0029s + 0.00073219,$$

$$a_{p21} = -11.72s + 0.18730,$$

$$a_{p22} = s^3 + 2.6542s^2 + 0.29166s + 0.39591$$

$$b_{p12} = -0.17300,$$

$$b_{p21} = -31.604s - 0.67310.$$

For α control, the controller can be PI control, because a_{11} is the first order. With the actuator dynamics, the controller is shown as follows.

$$(l_2s^2 + s)\delta_c^* = (k_1s + k_0)(\alpha_r - \alpha), \quad (19)$$

$$l_2 = 1/30.$$

Then the open-loop transfer function $G(s)$ becomes

$$G(s) = \frac{-0.17300(k_1s + k_0)}{(s^2/30 + s)(s + 1.9876)}. \quad (20)$$

The characteristic polynomial $P(s)$ becomes

$$P(s) = (s^2/30 + s)(s + 1.9876) - 0.17300(k_1s + k_0). \quad (21)$$

Now the crossover frequency is chosen as 15 rad/sec, one half of the actuator dynamics 30 rad/sec. By this choice,

$$k_1 = 15/(-0.17300) = -86.705. \quad (22a)$$

The stability index γ_1 is chosen as 4. The standard choice is 2.5, but this value is increased with the consideration of the effects of skew terms and of the sensitivity specification. Then k_0 is obtained as follows:

$$k_0 = (15 + 1.9876)^2 / [-0.17300 * 4 * (1 + 1.9876/30)] = -391.11. \quad (22b)$$

The coefficient diagram is shown in Fig. 3. In the similar manner, θ control is designed. The controller takes the following form.

$$(l_3^*s + 1)(l_2^*s^2 + s)\delta_e^* = (k_2^*s^2 + k_1^*s + k_0^*)(\theta_r - \theta),$$

$$l_3^* = 0.003, \quad l_2^* = l_2 = 1/30. \quad (23)$$

The value of l_3^* is more or less arbitrarily selected for pseudo-differentiation. Then the open-loop transfer function $G(s)$ becomes

$$G(s) = \frac{(-31.604s - 0.67310)(k_2^*s^2 + k_1^*s + k_0^*)}{A_l(s)},$$

$$A_l(s) = (l_3^*s + 1)(l_2^*s^2 + s)a_{p22}, \quad (24)$$

$$= 0.0001s^6 + 0.036599s^5 + 1.0965s^4$$

$$+ 2.6648s^3 + 0.30604s^2 + 0.39591s.$$

The characteristic polynomial $P(s)$ becomes

$$P(s) = A_l(s) \quad (25)$$

$$+ (-31.604s - 0.67310)(k_2^*s^2 + k_1^*s + k_0^*).$$

By the similar choice of the crossover frequency,

$$k_2^* = 15/(-31.604) = -0.47462. \quad (26a)$$

The rest of parameters are obtained, by specifying the stability index, $\gamma_3 = 4$ and $\gamma_2 = 2.5$. The γ_1 can not be specified, because of the small zero of the plant.

$$k_1^* = -2.2315, \quad k_0^* = -3.5669 \quad (26b)$$

The coefficient diagram is shown in Fig. 4. Due to the small zero in the plant, the coefficient of the 0-th order can not be designed and has to be accepted as it

is. Finally the total controller is expressed as follows by Eq. (14):

$$A_c(s) E^{-1} [u_e; u_c] = B_c(s) [\alpha_r - \alpha; \theta_r - \theta], \quad (27)$$

$$A_c(s) = \begin{bmatrix} a_{c11} & 0 \\ 0 & a_{c22} \end{bmatrix}, \quad B_c(s) = \begin{bmatrix} 0 & b_{c12} \\ b_{c21} & 0 \end{bmatrix},$$

$$a_{c11} = l_3^* s^2 + s, \quad a_{c22} = s,$$

$$b_{c12} = k_2^* s^2 + k_1^* s + k_0^*, \quad b_{c21} = k_1 s + k_0,$$

$$k_1 = -86.705, \quad k_0 = -391.11, \quad l_3^* = 0.003$$

$$k_2^* = -0.47462, \quad k_1^* = -2.2315, \quad k_0^* = -3.5669,$$

$$E = \begin{bmatrix} 1.3730 & 0.97298 \\ 0.52638 & 1.3730 \end{bmatrix}.$$

The controller is a 3-rd order controller consisting of one PI and one PID controller.

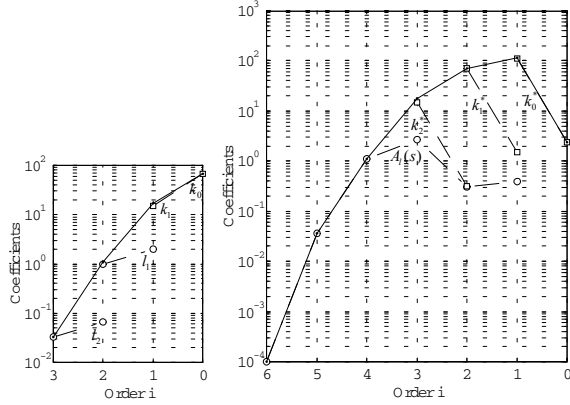


Fig. 3. α control

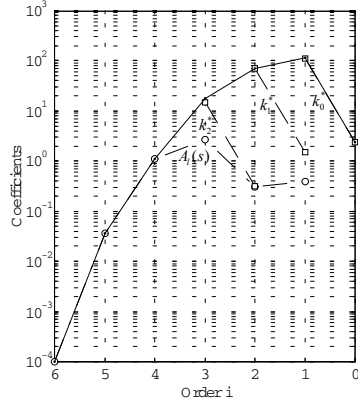


Fig. 4. θ control

6. SIMULATION

Simulation is performed with the designed controller Eq. (27) and the exact fighter model Eq. (9). Two cases are shown. The first case, Fig. 5, is for α_r step command, and the second case, Fig. 6, is for θ_r step command. In both cases, the outputs follow the reference commands, but in θ_r step case, the cross coupling effect is larger and α is appreciably affected. This suggests the interaction problem is still to be solved.

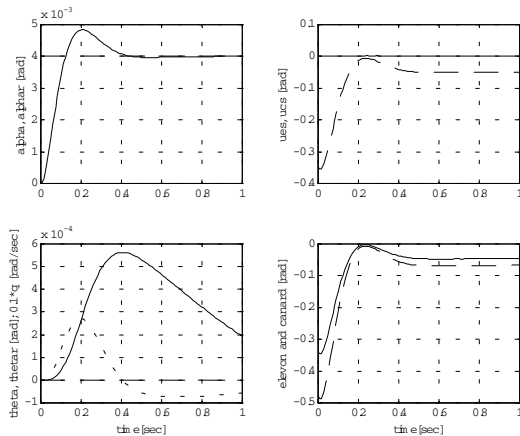


Fig. 5. α_r step command response

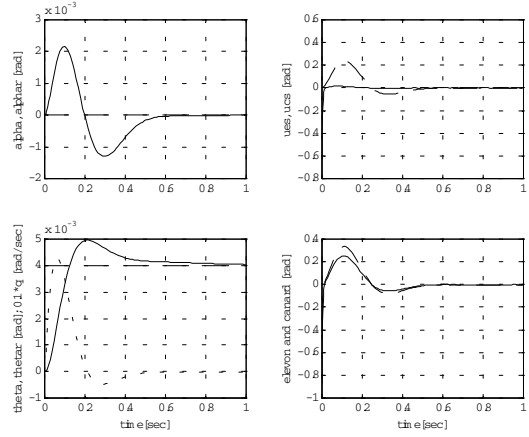


Fig. 6. θ_r step command response

7. COMPARISON WITH H-INF CONTROLLER

In order to make comparison with the H-inf controller, the sensitivity function $S(s)$ and auxiliary sensitivity function $T(s)$ for this controller is shown in Fig. 7. The definitions are as follows:

$$[\alpha; \theta] = T(s) [\alpha_r; \theta_r], \quad (28)$$

$$[\alpha_r - \alpha; \theta_r - \theta] = S(s) [\alpha_r; \theta_r],$$

$$T(s) = [T_{11} \quad T_{12}; T_{21} \quad T_{22}],$$

$$S(s) = [S_{11} \quad S_{12}; S_{21} \quad S_{22}].$$

Naturally, the following relations hold.

$$T_{11} + S_{11} = T_{22} + S_{22} = 1,$$

$$T_{12} + S_{12} = T_{21} + S_{21} = 0. \quad (29)$$

The singular value plots are shown in Fig. 8. They correspond to the worst case of Fig. 7. The svT1, larger singular value of $T(s)$, is greatly affected by the interaction term, T_{12} . The svS1, the larger value of $S(s)$, is greatly affected by $S_{12} = -T_{12}$, and slightly affected by $S_{21} = -T_{21}$ at the low frequency range. Thus if T_{12} and T_{21} are made small by some effective interaction minimization control, The singular value plot will become more favorable.

The achieved gamma value for the 8-th order H-inf controller in the Robust Control Toolbox is 16.8. The corresponding gamma value for this CDM 3-rd order controller is about 7 as read from Fig. 7. When proper interaction suppression is achieved, this gamma value will go up to about 14 with a large margin left to the robustness specification for $T(s)$ at 100 rad/sec. From this observation and the reported sigma plot, the 8-th order H-inf controller has good interaction suppression characteristics.

Even if the optimization is ideally made, the gamma value may not be able to go up beyond 22.361 ($=10^5 \cdot 5^{0.5}$), as easily guessed by a crude Bode diagram analysis supported by CDM. The key issue is to find a good interaction suppression control law. If such control law is found, gamma value may go up

close to 22.361. Whether such control does exist or not is an open question to be left for future studies.

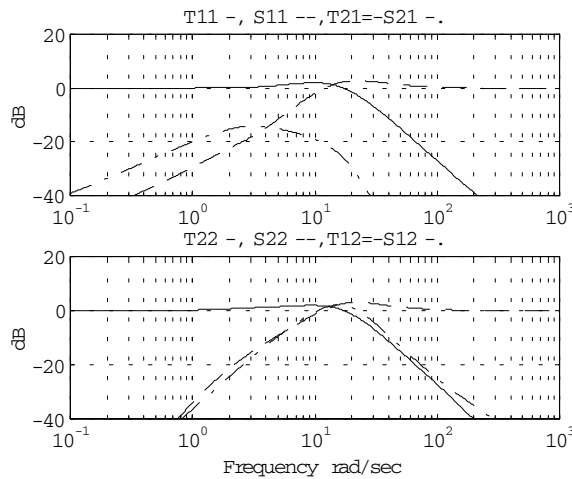


Fig. 7. Sensitivity and auxiliary sensitivity function

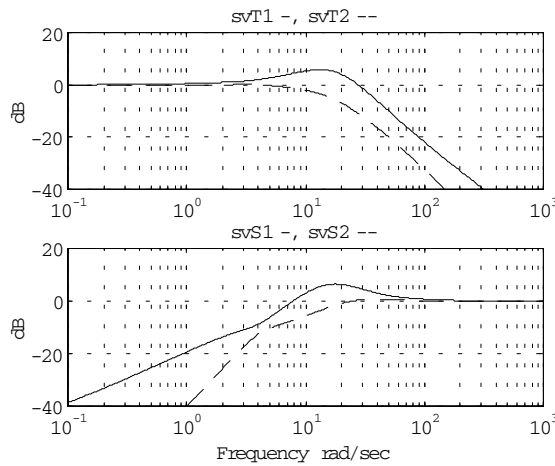


Fig. 8. Singular value plot

8. CONCLUSION

The major results of this paper are as follows:

- (1) A simple controller is designed for the longitudinal control of a modern fighter with elevon and canard.
- (2) The designed controller is a 3rd order controller composed of one PI and one PID. It achieves about a half of γ achieved by the 8-th order controller designed by H-inf.
- (3) In order for CDM to reach to the maturity in MIMO design, the suppression of the interaction is essential. The future studies in theory and algorithm in this respect are keenly needed.
- (4) The future development of effective CAD for MIMO CDM is keenly needed. The present CDM CAD (MSS, 2000) is very useful for many CDM design problems, but lacks in flexibility for use in MIMO design.

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