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Recent Development of Coefficient Diagram Method

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Abstract

A controller design method, called Coefficient Diagram Method (CDM), is introduced. By this method the simplest controller to satisfy the specification can be designed efficiently. The designer can design the controller and the characteristic polynomial of the closed-loop system simultaneously taking a good balance of stability, response, and robustness.

Keywords: Control system design, control theory, controllers, stability, polynomials.

1. Introduction

With wide spread of control technology into various fields, simple and reliable control design approach is keenly needed. The classical control well answered to this need for the ordinary control design problems, but not for more complex plants. The modern control has been developed to answer to this need. But it has not reached to the satisfactory state, because of complexity of the theory, unnecessarily high order of the designed controller, difficulty in tuning, and lack of robustness.

The Coefficient Diagram Method (CDM) has been developed to answer this problem. The CDM is fairly new and it is not well-known, but its basic philosophy has been known in industry and in control community for more than 40 years [1][2][4][14] with successful application in servo control [1], steel mill drive control [4], gas turbine control [12], and spacecraft attitude control [7]. The historical background is given in [10].

In Section 2, basic philosophy of CDM is explained to give the total picture of CDM. In Section 3, basics of CDM are explained, and in Section 4 recent development of CDM design process is briefly explained. Examples are not shown in this paper because of space limitation.

2. Basic philosophy of CDM

All the control system design for linear time invariant dynamic systems boils down to proper selection of the characteristic polynomials (denominator of transfer functions) and proper selection of the numerator polynomials for concerned input-output relations. When these polynomials are selected, the design of controller transfer function is straight forward, and requires only simple mathematics, when powerful computational tools are freely available as is today.

Especially the proper selection of the characteristic polynomial is essential in designing a good control system with proper balance of stability, response, and robustness. In any control system, the controller has limitation. The controller should be low order, minimum phase if possible, and stable unless unstable controller is absolutely necessary. The controller has a bandwidth limitation and power limitation in practice. These limitations impose strict limitation at the choice of characteristic polynomial. If characteristic polynomial is chosen without this consideration, the system usually looses robustness, although stability and response requirements are satisfied. When the plant is complex and difficult to control, stability and robustness become a trade-off issue, and one is to be sacrificed for the other.

The CDM gives the way to directly design the characteristic polynomial under controller limitation. The essence of CDM is "Coefficient Diagram", "Sufficient condition of stability by Lipatov", and "Improved version of Kessler standard form". The coefficient diagram is a semi-log diagram where the coefficients of characteristic polynomial are shown in the ordinate in logarithmic scale and the numbers of powers corresponding to the coefficients are shown in the abscissa in linear scale.

As Bode diagram and Nyquist diagram give information on stability and response, the coefficient diagram gives information on stability, response, and robustness. The convexity of the curve is a measure of stability. The general inclination of the curve is a measure of response speed. The variation of the shape of the curve due to plant/controller parameter variation is a measure of robustness.

Because the coefficients of the characteristic polynomial is related to the plant and controller parameters in closed form (usually in linear combination), design under controller limitation with robustness consideration becomes possible.

The design of characteristic polynomial in an efficient manner necessitates to express the plant and controller in polynomials, not transfer function nor state space expression. By so doing, the ambiguity inherent to transfer function (pole zero cancellation) and complexity of expression in state space (not n, but n^2 parameters in A matrix) are avoided.

The sufficient condition of stability by Lipatov is least known in the control community, but it is the theoretical basis of CDM. It gives the limit of convexity of the curve for the system to be stable.

The improved version of Kessler standard form gives the optimum convexity of the curve, which the designer is to seek under the constraint of controller limitation and robustness requirement.

When the characteristic polynomial is designed, the controller design is the same as in the pole assignment approach. It is interesting to note that, when the CDM design result is properly interpreted, the weights of LQR can be obtained, where the order of observer can be lower than the reduced-order observer [9].

3. Basics of CDM

3.1 Mathematical model

The standard block diagram of the CDM design for a single-input single-output system is shown in Fig. 1. The plant equation is given as

$$A_p(s)x = u + d \qquad (1a)$$

$$y = B_p(s)x\,, \qquad (1b)$$

where u, y, and d are input, output, and disturbance. The symbol x is called the basic state variable. $A_p(s)$ and $B_p(s)$ are the denominator and numerator polynomial of the plant transfer function $G_p(s)$. It will be easily seen that this expression has a direct correspondence with the control canonical form of the state-space expression, and x corresponds to the state variable of the lowest order. All the other states are expressed as the derivatives of x of high order.

Controller equation is given as

$$A_c(s)u = B_a(s)y_r - B_c(s)(y+n)\,, \qquad (2)$$

where y_r and n are reference input and noise on the output. $A_c(s)$ is the denominator of the controller transfer function. $B_a(s)$ and $B_c(s)$ are called the reference numerator and feedback numerator of the controller transfer function. Because the controller transfer function has two numerators, it is called two-degree-of-freedom system. This expression corresponds to the observer canonical form of the state-space expression.

Elimination of y and u from Eq. (2) by Eqs. (1a, b) gives

$$P(s)x = B_a(s)y_r + A_c(s)d - B_c(s)n\,, \qquad (3a)$$

where $P(s)$ is the characteristic polynomial and given as

$$P(s) = A_c(s)A_p(s) + B_c(s)B_p(s)\,. \qquad (3b)$$

In a similar manner, equation for y and u can be obtained. Because this system has 3 inputs and 3 outputs, there are 9 transfer functions.

For CDM design, the following four basic relations are selected as standard, namely

$$P(s)x = P(0)y_r \qquad (4a)$$

$$P(s)y = B_p(s)B_a(s)y_r \qquad (4b)$$

$$P(s)y = B_p(s)A_c(s)d \qquad (4c)$$

$$P(s)(-y) = B_p(s)B_c(s)n\,. \qquad (4d)$$

Eq. (4a) is the response of x to y_r when $B_a(s) = P(0)$, and it corresponds to the canonical closed-loop transfer function of system type 1 for $P(s)$, which will be explained later. This equation specifies the characteristic polynomial, and it is a very good measure of stability. Eq. (4b) is for the command following characteristics. Eq. (4c) is for the disturbance rejection characteristics. Eq. (4d) corresponds to the complementary sensitivity function $T(s)$, and it is useful for checking the robustness. In the CDM design, these four basic relations are used as performance specification. The design of $P(s)$ is first made to satisfy specifications on Eqs. (4a)(4c)(4d), and then $B_a(s)$ is adjusted to satisfy the specification on Eq. (4b).

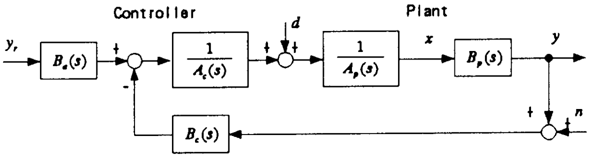

Fig. 1. Mathematical model

3.2 Mathematical relations

Some mathematical relations extensively used in CDM will be introduced hereafter. The characteristic polynomial is given in the following form.

$$P(s) = a_n s^n + \ldots + a_1 s + a_0 = \sum_{i=0}^{n} a_i s^i \qquad (5)$$

The stability index γ_i, the equivalent time constant τ, and stability limit γ_i^* are defined as follows.

$$\gamma_i = a_1^2/(a_{i+1}a_{i-1})\,, \quad i = 1 \sim n-1 \qquad (6a)$$

$$\tau = a_1/a_0 \qquad (6b)$$

$$\gamma_i^* = 1/\gamma_{i+1} + 1/\gamma_{i-1}$$
$$i = 1 \sim n-1, \quad \gamma_n = \gamma_0 = \infty \qquad (6c)$$

Also the equivalent time constant of the i-th order τ_i is defined as follows.

$$\tau_i = a_{i+1}/a_i\,, \qquad i = 1 \sim n-1 \qquad (7a)$$

Then the following relations are derived.

$$\tau_i = \tau_{i-1}/\gamma_i = \tau/(\gamma_i \cdots \gamma_2\gamma_1) \qquad (7b)$$

$$a_i = \tau_{i-1} \ldots \tau_2\tau_1\tau \;\; a_0$$
$$= a_0\tau^i / \;\; (\gamma_{i-1}\gamma_{i-2}^2 \cdots \gamma_2^{i-2}\gamma_1^{i-1}) \qquad (7c)$$

Then characteristic polynomial will be expressed by a_0, τ and γ_i as follows.

$$P(s) = a_0[\{\sum_{i=2}^{n}(\prod_{j=1}^{i-1} 1/\gamma_{i-j}^{\,j})(\tau s)^i\} + \tau s + 1] \qquad (8)$$

The stability index of the j-th order $\gamma_{i\,j}$ is defined as follows.

$$\gamma_{ij} = a_i^2/(a_{i+j}a_{i-j}) = [\prod_{k=1}^{j-1}(\gamma_{i+j-k}\gamma_{i-j+k})^k]\gamma_i^j \quad (9a)$$

Thus τ can be considered the equivalent time constant of the 0-th order and γ_i is considered as the stability index of the 1st order. The stability index of the 2nd order is a good measure of stability and is shown below.

$$\gamma_{i2} = a_i^2/(a_{i+2}a_{i-2}) = \gamma_{i+1}\gamma_i^2\gamma_{i-1} \quad (9b)$$

3.3 Coefficient diagram

When the plant/controller polynomials are given as

$$A_p = 0.25s^4 + s^3 + 2s^2 + 0.5s, \quad B_p(s) = 1,$$

$$A_c(s) = l_1 s, \quad B_c(s) = k_2 s^2 + k_1 s + k_0, \quad (10)$$

$$l_1 = 1, \quad k_2 = 0.5, \quad k_1 = 1, \quad k_0 = 0.2,$$

the characteristic polynomial is expressed as

$$P(s) = 0.25s^5 + s^4 + 2s^3 + 2s^2 + s + 0.2. \quad (11a)$$

Then

$$a_i = [a_5 \cdots a_2\ a_1] = [0.25 \quad 1 \quad 2 \quad 2 \quad 1 \quad 0.2] \quad (11b)$$

$$\gamma_i = [\gamma_4 \cdots \gamma_2\ \gamma_1] = [2 \quad 2 \quad 2 \quad 2.5] \quad (11c)$$

$$\tau = 5 \quad (11d)$$

$$\gamma_i^* = [\gamma_4^* \cdots \gamma_2^*\ \gamma_1^*] = [0.5 \quad 1 \quad 0.9 \quad 0.5] \quad (11e)$$

The coefficient diagram is shown as in Fig. 2, where coefficient a_i is read by the left side scale, and stability index γ_i, equivalent time constant τ, and stability limit γ_i^* are read by the right side scale. The τ is expressed by a line connecting 1 to τ. The stability index γ_i can be graphically obtained (Fig. 3a). If the curvature of the a_i becomes larger (Fig. 3a), the system becomes more stable, corresponding to larger stability index γ_i. If the a_i curve is left-end down (Fig. 3b), the equivalent time constant τ is small and response is fast. The equivalent time constant τ specifies the response speed.

The coefficient diagram is also used for parameter sensitivity analysis and robustness analysis. In this example, the characteristic polynomial $P(s)$ is decomposed into two component polynomials as follows.

$$P(s) = P_{l1}(s) + P_k(s) \quad (12a)$$

$$P_{l1}(s) = l_1(0.25s^5 + s^4 + 2s^3 + 0.5s^2) \quad (12b)$$

$$P_k(s) = k_2 s^2 + k_1 s + k_0 \quad (12c)$$

The auxiliary sensitivity function T(s) is expressed as

$$T(s) = P_k(s)/P(s) \quad (12d)$$

Eq. (12b) is shown in Fig. 2 with small circles and dotted lines. Eq. (12c) is shown with small squares and dotted lines. Designer can visually assess the deformation of the coefficient diagram due to the parameter change of k_2, k_1, and k_0. Then he can visualize the variation of stability and response. Also from Eq. (12d), it is clear that robustness can be analyzed by comparison of coefficients a_i and k_i at the coefficient diagram.

As explained above, the coefficient diagram indicated stability, response, and robustness (three major properties in control design) in a single diagram, enabling the designer to grasp the total picture of control system. At present, Bode diagram is used for this purpose. However coefficient diagram is more accurate and easy to use in actual design.

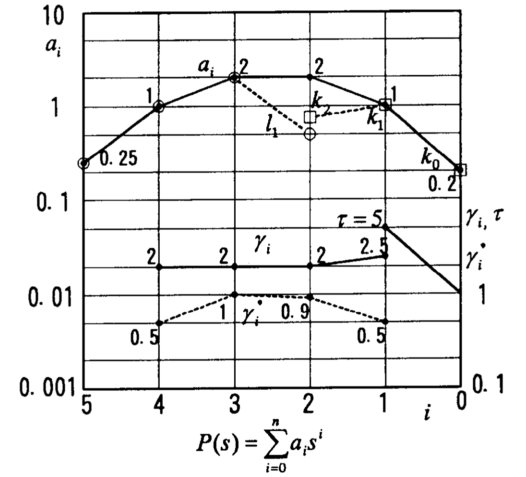

Fig. 2. Coefficient diagram

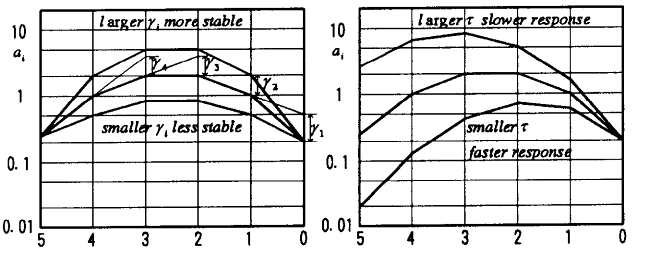

Fig. 3a. Effect of γ_i b. Effect of τ

3.4 Stability condition

From the Routh-Hurwitz stability criterion, the stability condition for the 3rd order is given as

$$a_2 a_1 > a_3 a_0 \quad . \quad (13a)$$

If it is expressed by stability index,

$$\gamma_2\gamma_1 > 1. \quad (13b)$$

The stability condition for the fourth order is given as

$$a_2 > (a_1/a_3)a_4 + (a_3/a_1)a_0 \quad (14a)$$

$$\gamma_2 > \gamma_2^*. \quad (14b)$$

For the system higher than or including 5th degree, Lipatov [5] gave the sufficient condition for stability and instability in several different forms. The conditions most suitable to CDM can be stated as follows;

"The system is stable, if all the partial 4th order polynomials are stable with the margin of 1.12. The system is unstable if some partial 3rd order polynomial is unstable."

Thus the sufficient condition for stability is given as

$$a_i > 1.12[\frac{a_{i-1}}{a_{i+1}}a_{i+2} + \frac{a_{i+1}}{a_{i-1}}a_{i-2}] \quad (15a)$$

$$\gamma_i > 1.12\gamma_i^*, \quad for\ all\ i = 2 \sim n-2. \quad (15b)$$

The sufficient condition for instability is given as

$$a_{i+1}a_i \le a_{i+2}a_{i-1} \quad (16a)$$

$$\gamma_{i+1}\gamma_i \le 1, \quad for\ some\ i = 1 \sim n-2. \quad (16b)$$

These conditions can be graphically expressed in the coefficient diagram. Fig. 4a is a 3rd-order example. Point A is $(a_2\, a_1)^{0.5}$ and point B is $(a_3\, a_0)^{0.5}$. Thus if A is above B, the system is stable. Point C is $(\gamma_2\, \gamma_1)^{0.5}$. If it is above 1, the system is stable.

Fig. 4b is a 4th-order example. Point A is obtained by drawing a line from a_4 in parallel with line $a_3\, a_1$. Similarly point B is obtained by drawing a line from a_0 in parallel with line $a_3 a_1$. The stability condition is $a_2 >$ (A + B). The other condition is $\gamma_2 > \gamma_2^*$.

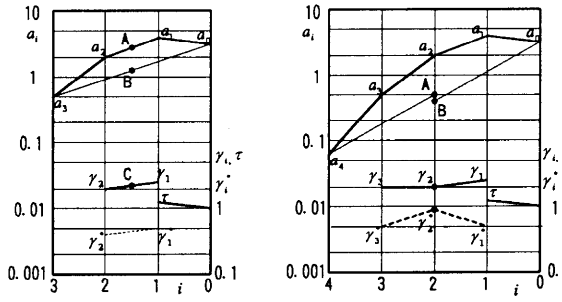

Fig. 4a. 3rd order b. 4th order

3.5 Canonical transfer function

For a given characteristic polynomial, there exist infinite number of open-loop and closed-loop transfer functions. The specific transfer functions to represent the characteristic polynomial, called canonical transfer function, are defined as follows.

System type 1, canonical open/closed-loop transfer function, $G_1(s)$ and $T_1(s)$.

$$G_1(s) = a_0 / (a_n s^n + \cdots + a_1 s) \tag{17a}$$

$$T_1(s) = a_0 / (a_n s^n + \cdots + a_1 s + a_0) \tag{17b}$$

System type 2, canonical open/closed-loop transfer function, $G_2(s)$ and $T_2(s)$.

$$G_2(s) = (a_1 s + a_0) / (a_n s^n + \cdots + a_2 s^2) \tag{17c}$$

$$T_2(s) = (a_1 s + a_0) / (a_n s^n + \cdots + a_1 s + a_0) \tag{17d}$$

These canonical open/closed-loop transfer functions are uniquely defined by the characteristic polynomial $P(s)$, and they are helpful to visualize the characteristics of $P(s)$.

Also break point ω_i is defined as

$$\omega_i = a_i / a_{i+1} = 1/\tau_i \,. \tag{18a}$$

The ω_i is the reciprocal of the equivalent time constant of high order τ_i. The ratio of adjacent break points is equal to the stability index γ_i.

$$\gamma_i = \omega_i / \omega_{i-1} \tag{18b}$$

Fig. 5 shows an example of Bode diagram of the canonical open-loop transfer function for the system type 1 and 2. The straight-line approximation (asymptotic representation) of Bode diagram used here is somewhat different from the ordinary way. The break points are chosen from the ratio of the coefficients and not from the poles and zeros of the transfer function as in the usual case. However this way is more accurate and the relation with the coefficient diagram is closer.

Thus it becomes clear that the coefficient diagram has a one-to-one correspondence with the straight-line approximation of Bode diagram of its canonical open-loop transfer function.

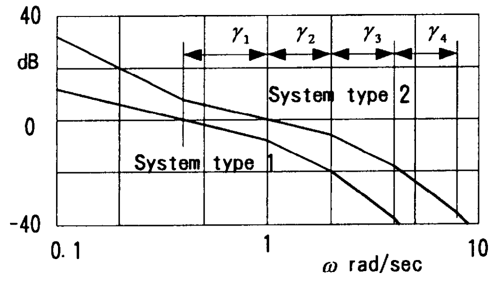

Fig. 5. Canonical open-loop transfer function

3.6 Standard form

From number of reasons, which will become clear later, the recommended standard form for CDM is

$$\gamma_{n-1} \sim \gamma_2 = 2, \qquad \gamma_1 = 2.5\,. \tag{19a}$$

When $a_0 = 0.4$ and $\tau = 2.5$ are chosen, the characteristic polynomial $P(s)$ is obtained by Eq. (7c) in the following simple form

$$P(s) = 2^{-\frac{(n-2)(n-1)}{2}} s^n + \cdots + 2^{-10}s^6 + 2^{-6}s^5 + 2^{-3}s^4 + 0.5s^3 + s^2 + s + 0.4 \tag{19b}$$

The step response of the canonical closed-loop transfer function for the system type 1 and 2 for various orders are given in Fig. 6 and 7. There is virtually no overshoot for the system type 1. There is an overshoot of about 40% for system type 2. This overshoot is necessary, because the integral of the error for the step response must become zero in system type 2. It is also noticed that the responses are about the same irrespective to the order of the system. Because of this nature, the designer can start from a simple controller and move to more complicated one in addition to the previous design. The settling time is about 2.5 ~ 3τ. Many simulation runs show that the standard form has the shortest settling time for the same value of τ.

The pole location is given in Fig. 7. It is found that the three lowest order poles are aligned in a vertical line and the two highest order poles are at the point about 49.5 deg from the negative real axis. The rest of the poles are on or close to the negative real axis. For 4th order, all poles are exactly on the vertical line.

It can be mathematically proven that a 3rd order system with three poles on a vertical line has no overshoot. For γ_2=2 and γ_1=2.7, three poles are on a vertical line and overshoot is zero. If γ_1 = 2.5 as in the standard form, the complex poles are a little bit closer to the imaginary axis with the result of a small overshoot. The choice of γ_1 = 2.5 instead of 2.7 is made for the reason of simplicity.

In summary, the standard form has the favorable characteristics as listed below.

(1) For system type 1, overshoot is almost zero. For system type 2, necessary overshoot of about 40 % is realized.

(2) Among the system with the same equivalent time constant τ, the standard form has the shortest settling time. The settling time is about $2.5\sim3\tau$.
(3) The step responses show almost equal waveforms irrespective to the order of the characteristic polynomials.
(4) The lower order poles are aligned on a vertical line. The higher order poles are located within a sector 49.5 degrees from the negative real axis, and their damping ratio ζ is larger than 0.65.
(5) The CDM standard form is very easy to remember.

In other words, the standard form seems to posses all the characteristics of "good designs" found from experience, such as no overshoot, short settling time, and pole alignment on a vertical line. For comparison, stability indices γ_i's for various standard forms used in the control theory are given in Table 1. It is found that CDM standard is similar to Bessel at the low order, and become similar to binomial at the high order.

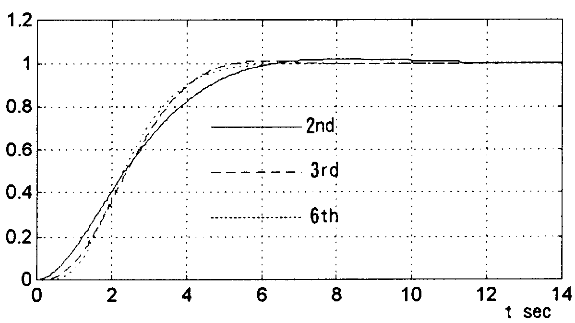

Fig. 6. Step response, system type 1

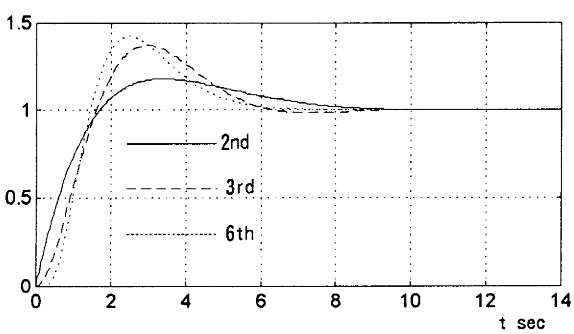

Fig. 7. Step response, system type 2

3.7 Robustness consideration

Robustness and stability are completely different concepts. Simply stated, stability concerns where the poles are located, and robustness concerns how fast the poles move to the imaginary axis for the variation of parameters.

Stability is specified by the stability index γ_i of the characteristic polynomial, but robustness is only specified after the open-loop structure is specified. Thus in designing the characteristic polynomial, more consideration is required beyond the choice of γ_i. The traditional design principle of sticking to the minimum-phase controller, wherever possible, with the lowest

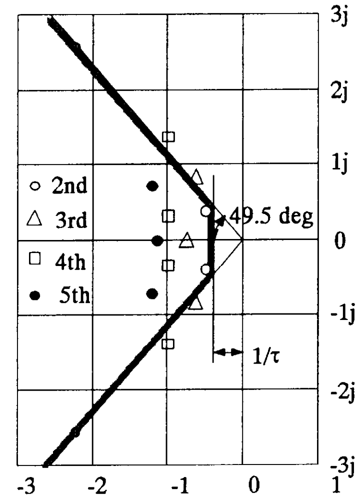

Fig. 8. Pole location

Table 1. Comparison of stability index

Standard forms	Stability index γ_4	γ_3	γ_2	γ_1	Standard forms	Stability index γ_4	γ_3	γ_2	γ_1
Binomial				4	ITAE				2
			3	3				1.424	2.641
		2.667	2.25	2.667			1.297	2.039	2.144
	2.5	2	2	2.5		1.568	1.624	1.779	2.102
Butter-worth				2	Kessler				2
			2	2				2	2
		2	1.707	2			2	2	2
	2	1.618	1.618	2		2	2	2	2
Bessel				3	CDM				2.5
			2.4	2.5				2	2.5
		2.222	1.929	2.333			2	2	2.5
	2.143	1.75	1.778	2.25		2	2	2	2.5

possible order and with the narrowest possible bandwidth is actually found to be a strong guarantee of robustness.

In the actual design, the choice of $\gamma_1 = 2.5$, $\gamma_2 = \gamma_3 = 2$ is strongly recommended due to stability and response requirement, but it is not necessary to make $\gamma_4 \sim \gamma_{n-1}$ equal to 2. The condition can be relaxed as

$$\gamma_i > 1.5\gamma_i^* \tag{20}$$

With such freedom, the designer has the freedom of designing the controller together with the characteristic polynomial, and he can integrate robustness in the characteristic polynomial with a small sacrifice of stability and response. From the sufficient condition for stability by Lipatv, stability is guaranteed when all γ_i's are larger than 1.5. Lipatov proved in his paper [5] that, if all γ_i's are greater than 4, all the roots are negative real. Thus γ_i's are usually chosen between 1.5 and 4. Because the essence of the CDM lies in the proper selection of stability indices γ_i's, some experiences are required in actual design, as is true in any design effort.

4. Recent Development

There have been three stages in the CDM development.
(1) At the first stage, design was made only by stability indices γ_i's. When stability indices are specified, the controller parameters can be expressed in a closed form. But they are usually in the form of

nonlinear simultaneous equations, and difficult to solve except for the simple case.

(2) At the second stage, the coefficients of the characteristic polynomial a_i's are first calculated from the specified stability indices and the equivalent time constant τ. Then the controller parameters are calculated. The controller parameters are related to a_i's in linear relations called Sylvester matrix, and solution becomes straightforward. But this method is equivalent to the pole assignment approach. Thus if it is not wisely done, robustness issue will come up.

(3) At the third stage, the coefficient diagram is more actively used. This becomes possible by the discovery of the graphical representation, in the coefficient diagram, of the sufficient condition of stability by Lipatov. With some experience, designer can answer such problems as "What is the proper order of the controller suitable to the plant and the performance requirement" or "What degree of stability and robustness tradeoff exists" from the coefficient diagram. Special CAD for CDM was developed on MATLAB basis. It can be downloaded from the following site.

http://www.mss.co.jp/techinfo/cdm-cad/index.htm

At present, the most efficient way of CDM design process is as follows. When the plant is given, the first step is to express it in the CDM standard block diagram. The second step is to assume denominator of controller in an appropriate simple form, usually 1 or s, and draw the coefficient diagram for no feedback.

The third step is the basic design of controller. From the shape of the coefficient diagram, the designer can intuitively find what type of feedback is necessary to modify the coefficient diagram into nice convex form. He can roughly sketch the final shape of coefficient diagram, and specify the degree of controller and the possible equivalent time constant.

The forth and final step is to design the complete controller by CAD for CDM. Parameter adjustment is done with the roughly sketched coefficient diagram as the design guide.

5. Conclusions

The CDM has developed on the needs and experiences of the actual controller design, and it contains the rich knowledge of the past designers about what is the good controller. For this reason the design result is reliable and needs minimum amount of field adjustment.

The CDM can be used as an independent design approach, but at the same time it can be used to help other approaches, such as the parameter selection of a lead-lag compensator, PID parameter selection, and weights selection of LQG. Also the CDM can be used for the evaluation of the controllers designed by other approaches, simply by drawing the coefficient diagram of the design results.

The CDM as explained so far is at the stage of development and further effort is needed to make it to full maturity. At the present stage, only SISO (including SIMO and MISO) system is considered. The extension to MIMO system is left for the future studies.

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